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A reinsurance and investment game between two insurance companies with the different opinions about some extra information ☆

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ABSTRACT

The work studies a reinsurance and investment game between two insurance companies which have different opinions about some extra information. We assume that the goal of each insurance company is to maximize its utility of the difference between its terminal surplus and that of its competitor at the terminal time T. Moreover, at the beginning of the game, two insurance companies acquire some information about the future realization of the claims process. However, they treat with it differently, since one company trusts it while its competitor does not. Our focus is to study how the companies are affected by this information. By utilizing the dynamic programming principle and the enlargement of filtration techniques, the existence of the Nash equilibrium solutions can be verified. For the exponential utility, we derive three kinds of the candidate forms for the equilibrium strategies in the special situations and also provide the numerical method for the general situation. Some numerical examples are presented to illustrate how the reinsurance strategies change when the information level and other parameters vary.

1. Introduction

Based on the pioneer works such as Bühlmann (1970), the stochastic optimal control plays a prominent role in actuarial study in recent years. In practice, the major insurance companies can enter reinsurance contracts to transfer parts of their risks to the reinsurance companies. At the same time, they have the opportunities to invest in the stock markets to diversify their risks and wealth. The insurers' strategies are decided according to the dynamic and stochastic market status. The study of optimal investment and reinsurance strategies in the framework of stochastic optimal control theory is an interesting and fruitful area, in which many papers have been published. See for example Browne (1995), Yang and Zhang (2005), Schmidli (2008), and references therein.

With the increasingly fierce market competition, the stochastic differential reinsurance/investment games have been studied extensively very recently. In Taksar and Zeng (2011), authors focus on a zero-sum game between two insurance companies which employ non-proportion reinsurance strategies in order to reduce risk of exposure. Bensoussan et al. (2014) study the relative performance of two insurance companies under a non-zero sum stochastic differential game framework. Meng et al. (2015) and Pun and Wong (2016) both develop this model and introduce the concepts of nonlinear risk control processes and the ambiguity aversion into it, respectively. Besides, Guan and Liang study the portfolio game between two DC pension funds in Guan and Liang (2016).

We notice an interesting economic phenomenon in the insurance market. Most of existing works in the actuarial literatures suppose that the insurers make their decisions relying on all the information generated by the events in the insurance business up to now, instead of the time in the future. In the real world, the companies in the insurance market have access to the different levels of the information, such as some additional information about future. For example, some insurers who sell the agriculture insurance acknowledge the information about the future drought or flood. Another example happens in the medical insurance market. Among all the companies, there is some news that the cost of treatment of one major disease will decrease in the future due to a breakthrough in the medicine technology (see Baltas et al., 2012 and Peng and Wang, 2016). However, such information may be not credible. Therefore, there are probably two types of the companies in the market. The first type trusts the extra information or has the ability to verify it, while the second type

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does not. Meanwhile, those companies compete with each other in the insurance market. Therefore, we think that it is worthwhile to study the influence of the extra information and the different opinions about it on all the insurers in the framework of the game theory. For more studies about the information of the game, we refer to Rasmusen (2006).

In this paper, we study a reinsurance and investment game between two companies affected by some extra information about the future. Inspired by Baltas et al. (2012), we use the future realization of the claims process of one company as the extra information. To reflect the real world, in which there are various attitudes to the information, we assume that the first company believes in the information while the second does not. We focus on the equilibrium strategies in this circumstance. The similarities and differences between our work and Pun and Wong (2016) are as follows. Both papers are concerned with the reinsurance/investment game under two different risk measures. However, the reason that they introduce two risk measures is to reflect the insurers' confidences on the reference model. In our model, the measure changes due to the leakage of some additional information. Technically, they accomplish the change of measure by adding the penalty functions into the utility functions, while we alter the original stochastic processes explicitly and introduce a new variable to indicate the stochastic drift part of the information level.

Moreover, we should mention the recent work (Peng and Wang, 2016) which also studies the effects of the extra information on the investment and reinsurance strategies of the insurance companies in the single-agent framework. In that paper, the stochastic processes involved with the insurance business and the financial market are the correlated jump diffusion processes and the extra information is involved with both of them, which is a more general setting compared with Baltas et al. (2012). The authors have adopted a new method called the forward integral to derive the first order condition of the optimal strategies under the criterion of logarithmic utility function. As they have pointed, although the generality of the stochastic processes is considered, other utility functions except the logarithmic utility cannot be used with this method. However, the positive condition of the wealth processes is very crucial for the logarithmic utility function, which is hardly guaranteed when we adopt the concept of relative performance. Therefore, we still follow the ideas of Baltas et al. (2012) instead of Peng and Wang (2016) in this paper. The general model of the investment/reinsurance game with the extra information deserves to be investigated further in the future study.

The novelties of the paper are as follows. First, we reveal the effects of the extra information on two insurers in the reinsurance/investment game. Their actions are both affected by the extra information level, including the initial level and the stochastic drift. The explanation is that their actions are correlated based on the competitive relationship, and hence the second insurer is affected by the information level indirectly. However, their heterogeneous beliefs in the information lead to the asymmetric impacts on them. The impact on the second one is limited based on its distrust of the extra information. As we know, this result is new compared with the existing literatures. Second, from the technical aspect, we provide a new method to solve the strategies for the CARA insurers although we still apply the enlargement of filtration techniques and the dynamic programming principle used in Baltas et al. (2012). The constraint that the reinsurance company should cover the non-negative proportion of the claims is set forcedly in Baltas et al. (2012), which is only suitable for the single-agent problem. In our paper, we divide the computation process into two steps to deal with the constraint. In the first step, we derive the analytical solutions in some special information levels by separating the two variables and solving the ordinary differential equations (ODEs). Then, in the second step, we introduce the standard finite difference method to solve the nonlinear partial differential equation (PDE) numerically to derive the Nash equilibrium strategies in the general case, in which the boundary condition of the PDE is the analytical results in the first step. In our opinion, the new idea arising in our paper can demonstrate the relevant economical phenomenon and enlighten the future work technically at the same time.

The rest of the paper are organized as follows. In Section 2, we briefly describe the reference model which is the stochastic differential game with two insurers. In Section 3, we introduce the extra information and two opinions about it. The forms of the game and the Nash equilibrium solutions are formulated. To study the Nash equilibrium solutions, we apply the dynamic programming principle to obtain a verification theorem for the corresponding (coupled) Hamilton–Jacobi–Bellman equations in Section 4. In Section 5, we study the solutions for the equilibrium reinsurance and investment strategies under the exponential utility function. Section 6 gives the numerical method for the general case and two numerical examples in order to illustrate our results and discusses the effects of parameters. Some conclusions are given in Section 7.

2. Reference model

In this section, we describe the reference model in details. We first define the risk and surplus process which contains the reinsurance and investment opportunities. Then, two competing insurance companies, which follow the above process, are introduced.

2.1. The surplus process with reinsurance and investment opportunities

In the classical diffusion Cramer–Lundberg model, the surplus of the insurer follows

$$\begin{aligned} dR(t) &= a\eta dt + bdB(t), \\ R(0) &= x, \end{aligned} \tag{2.1}$$

where $B(t)$ is the standard Brownian motion, a and b are the claim rate and volatility, and η is the safety loading. To reduce the underlying risks involved in the claims process, the insurer has an option to purchase a proportional reinsurance protection. The function $u(t) : R^+ \to [0, +\infty)$ stands for a proportional reinsurance strategy, which means that the reinsurance company will cover $100(1 - u(t))\%$ of the claims at time t, while the insurance company will cover the remaining $100u(t)\%$. A proportional reinsurance premium is paid continuously at the constant rate $(1+\theta)(1-u(t))a$, where θ is the safety loading of the proportional reinsurance and satisfies $\theta \geq \eta$. We assume that the value of $u(t)$ can be more than 1. The situation that $u(t) > 1$ means that the insurance company acquires new business, for example, to be reinsurer for other insurance companies (see Chen et al., 2010 for details). In this situation, the surplus process becomes

$$\begin{aligned} dR(t) &= [(1+\eta) - u(t) - (1+\theta)(1-u(t))]\, adt + u(t)bdB(t) \\ &= (\eta - \theta + \theta u(t))adt + u(t)bdB(t). \end{aligned}$$

In addition, the insurer has an opportunity to invest in the stock markets. The insurer can invest in a risk-free asset, denoted by $S_0(t)$ and a risky asset, denoted by $S_1(t)$. Their dynamics are

$$dS_0(t) = rS_0(t)dt, \tag{2.2}$$

$$dS_1(t) = \mu S_1(t)dt + \sigma S_1(t)dW(t), \tag{2.3}$$

where $W(t)$ denotes the standard Brownian motion, r denotes the risk-free interest rate, and μ and σ denote the return and volatility of the risky asset. The function $\pi(t) : R^+ \to [0, +\infty)$ represents