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The joint mortality of couples in continuous time

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ABSTRACT

This paper introduces a probabilistic framework for the joint survivorship of couples in the context of dynamic stochastic mortality models. The death of one member of a couple can have either deterministic or stochastic effects on the other; our new framework gives an intuitive and flexible pairwise cohort-based probabilistic mechanism that can account for both. It is sufficiently flexible to allow modelling of effects that are short-term (called the *broken-heart effect*) and/or long-term (named *life circumstances bereavement*). In addition, it can account for the state of health of both the surviving and the dying spouse and can allow for dynamic and asymmetric reactions of varying complexity. Finally, it can accommodate the pairwise dependence of mortality intensities before the first death. Analytical expressions for bivariate survivorship in representative models are given, and their sensitivity analysis is performed for benchmark cases of old and young couples. Simulation and estimation procedures are provided that are straightforward to implement and lead to consistent parameter estimation on synthetic dataset of 10000 pairs of death times for couples.

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1. Introduction

It is reasonable for most of us to accept the premise that the death of a spouse typically adversely affects the survivorship of the bereaved partner. While it is clear that there are a great many explanatory factors that shape the precise nature and intensity of this phenomenon, we can easily arrive at several plausible factors, such as the cohorts to which the partners belong, and the gender and level of health of the surviving partner. It also makes sense to allow this phenomenon to be time-dependent, with both short-term and permanent impacts.

Several empirical studies (see Denuit et al. (2001) or Jagger and Sutton (1991)) appear to confirm the common intuition of dependence between the lifetimes of members of a couple. However, according to Hougaard (2000), there are three aspects of this *bereavement effect* to distinguish. The first is the *common shock* effect, which accounts for simultaneous deaths of a couple from a common disaster (see Bowers (1997) and recently (Su and Furman, 2016)); the second is short-term dependence or the *broken-heart syndrome* (see Parkes et al. (1969) and Jagger and Sutton (1991)), which describes a period of higher mortality of an individual immediately after their partner's death, even though the causes of the two deaths may appear to be independent. The third phenomenon, long-term dependence, we will call *life circumstances bereavement*. In this work, we propose a framework which accounts for both short-term and long-term bereavement effects but not the common shock effect.

The bereavement phenomenon is worthwhile investigating from the perspective of both policy holders and issuers. For the issuers, a robust model for joint life survivorship would be highly beneficial for risk management of joint life insurance policies. The robust assessment of risks of this type in their internal models would encourage insurance companies to pass the savings to consumers, thus becoming more competitive. Given the current regulatory frameworks such as Solvency II in Europe and OSFI directives in Canada, such models would also reduce the capital holdings required.

Up until now, one finds two broad classes of joint mortality models in the literature. The first and larger class is a class of models starting from the works of Frees et al. (1996), Frees and Valdez (1998), Youn and Shemyakin (1999), Carriere (2000), Youn and Shemyakin (2001), Denuit et al. (2001), Luciano et al. (2008) and the work of Luciano et al. (2012). These models have in common copula-based dependence structures. The paper of Luciano et al. (2008) makes a connection for the first time between dependence structure derived by a copula approach and dynamic stochastic mortality modelling in continuous time. The second class of models is based on a Markov or semi-Markov chain modelling framework, historically spanned by the works of Norberg (1989), Spreeuw and Wang (2008), Ji et al. (2011) and Spreeuw and Owadally (2012).

Examples of stochastic mortality models of the single cohort type cast in continuous time are given by Milevsky and Promislow (2001); Dahl (2004); Biffis (2005); Cairns et al. (2006); Schrager

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(2006); Luciano and Vigna (2008) and Qian et al. (2010). Luciano et al. (2008) model the joint mortality experience for members of a couple with continuous time cohort models of affine type, as in Luciano and Vigna (2008), and impose dependence between lives by using copula functions. In our work, we similarly use stochastic mortality models of affine type for mortality experience of couples before the first death, but in contrast to Luciano et al. (2008) we develop a clear probabilistic mechanism for the impact of one life's death on the other. Moreover, our new approach allows for a natural and intuitive interpretation of short and long mortality dependence along the lines of Spreeuw and Owadally (2012). In addition, it can account for the state of health of both the surviving and the dying spouse and thus allow for dynamic and asymmetric reactions of varying complexity. Finally, it can accommodate the pairwise dependence of lives before the first death. Our modelling approach can be combined with existing mathematical finance techniques for pricing and hedging purposes, and hence has distinct comparative advantages over existing approaches.

The rest of this paper is organized as follows. In Section 2 we introduce the probabilistic framework and develop our main result and how it can be used. Section 3 focuses on the affine modelling setting, where we give a representative example called the Independent OU Exponential Decay Model (IOUED model). Section 4 reports the details of an exact sensitivity analysis for the probability density functions arising for two representative cases, a young and an old couple. Section 4 also includes a simulation procedure and a maximum likelihood estimation methodology. Section 5 concludes the paper. Some of the more detailed computations are reported in the Appendix.

2. A probabilistic framework

The aim of this paper is to propose a probabilistic framework to model the joint life expectancy of a couple or spousal pair, imagined as two adults cohabiting and thus sharing a common environment. Importantly, the mortality rate of the surviving partner will be assumed to increase after the first death in the couple. The proposed model will adapt the principles now known as *reduced form modelling* (RFM), as developed in financial mathematics for studying credit risk, the financial risk that a corporate borrower defaults on its obligations. Reduced form modelling is based on an intuitive picture that death occurs like a "bolt of lightning" whose actual timing is unpredictable, but with an instantaneous likelihood, or *intensity*, that is predictable.

2.1. Model setting

The model is formulated within a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where $\mathbb{P}$ is called the *real-world probability measure*. This probability space must be rich enough to support a d-dimensional Brownian motion W, random variables Z_1 and Z_2 having unit exponential distribution, and a single random variable U with the standard uniform distribution. The collection $\{W, Z_1, Z_2, U\}$ is a fully independent collection. The Brownian filtration $\mathcal{G}_t = \sigma(W_s), s \leq t$ is defined for $t \in [0, T]$ over a sufficiently long finite time horizon T.

We label the two partners of a couple as partner 1 and partner 2, and by τ_1 and τ_2 we denote their time of death. Let $p \in \{1, 2\}$ mark the identity of the partner who dies first, henceforth called the *deceased partner*, and q the identity of the surviving partner, called the *bereaved partner*. If $p = 1$ and $q = 2$ then the *first death time* is $\tau_p = \tau_1$ and the *second death time* is $\tau_q = \tau_2$. Similarly, if $p = 2$ and $q = 1$, then $\tau_p = \tau_2$ and $\tau_q = \tau_1$.

On the probability space, let us consider a pair of predictable $\mathcal{G}_t$-adapted processes λ_t^1 and λ_t^2 that prior to the first death time τ_p of a member of the couple represent the *instantaneous mortality intensities* of partner 1 and partner 2. In later sections, we will assume that mortality intensity processes are of so-called affine type (see Filipović (2009)) since they give an appropriate compromise between modelling flexibility and tractability.[1] The processes λ_t^1 and λ_t^2 are driven by W. Each realization of the random elements $\{W, Z_1, Z_2, U\}$ determines a realization of times of deaths of both members of a couple. The *first time of death* τ_p is defined to be

$$\tau_p := \inf \left\{ t \geq 0 \;\middle|\; \int_0^t (\lambda_u^1 + \lambda_u^2) du \geq Z_1 \right\}. \tag{2.1}$$

Next, the identity of the deceased partner is determined in terms of the trigger U as

$$\{p = 1\} = \{\tau_1 = \tau_p\} = \left\{ U \leq \frac{\lambda_{\tau_p}^1}{\lambda_{\tau_p}^1 + \lambda_{\tau_p}^2} \right\} \tag{2.2}$$

$$\{p = 2\} = \{\tau_2 = \tau_p\} = \left\{ U > \frac{\lambda_{\tau_p}^1}{\lambda_{\tau_p}^1 + \lambda_{\tau_p}^2} \right\}, \tag{2.3}$$

where $\lambda_{\tau_p}^1$ and $\lambda_{\tau_p}^2$ are the mortality intensities at the moment of the first death.

After the first death τ_p, the mortality intensity of the bereaved partner is assumed to be a modification $\tilde{\lambda}_t^q$ of λ_t^q where again the subscript q labels the bereaved partner. The relationship between $\tilde{\lambda}_t^q$ and λ_t^q will characterize the extra effect on one's health caused by the death of one's spouse. We call the surviving partner the bereaved partner to reflect the notion that the impact of the death of one's spouse on one's health is usually adverse. After the moment of the first death, the *adjusted mortality intensity process* of the bereaved partner is $\tilde{\lambda}^q$. The *change in mortality process* r^q defined as

$$r_t^q := \tilde{\lambda}_t^q - \lambda_t^q \tag{2.4}$$

can be regarded as a mathematical aggregation of all aspects of the bereavement effects.[2]

The process $\tilde{\lambda}_t^q$ is a modification of λ^q with an explicit structural break at time τ_p that reflects the direct effect on one's mortality rate that happens at the death of one's spouse. In each alternative, the time of death of the bereaved partner or the *second time of death* τ_q is determined conditionally by

$$\tau_q := \inf \left\{ t > \tau_p \;\middle|\; \int_{\tau_p}^t \tilde{\lambda}_u^q du \geq Z_2 \right\}. \tag{2.5}$$

Later in the paper we explore a specification of the processes r_t^q that leads to flexible mortality structures which admit tractable computations.

2.2. Using the model

It is important to understand at the outset how RFM extends to multiple individuals, or in our context to multiple couples, within a large population. This problem is analogous to *portfolio credit risk* where it has been found to have certain subtleties. First we append an index $n \in \{1, \ldots, N\} := [N]$ to each couple-specific element, including the collection $\{W^{(n)}, Z_1^{(n)}, Z_2^{(n)}, U^{(n)}\}_{n \in [N]}$ and the mortality intensity pairs $(\lambda^1, \lambda^2)^{(n)}$. The main issue to resolve is to specify dependence structures both within each couple, and across the population. For couple n, the d-dimensional Brownian motion $W^{(n)}$ determines the pair of intensities $(\lambda^1, \lambda^2)^{(n)}$. It has become common to view the correlation structure across the population as built up from *systematic risk factors* that are common

[1] By assuming that the probability space supports appropriate jump processes, the framework easily allows consideration of affine jump–diffusion processes.

[2] It is assumed that the probability space is rich enough to support this process.

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