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Confidence sets and confidence bands for a beta distribution with applications to credit risk management

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HIGHLIGHTS

- An exact confidence band for loss given default distribution in credit risk management is proposed.
- The approach based on a multiple comparison technique for a beta distribution.
- The resulting technique can be employed to rigorously stress test loss given default estimate with a limited data set.
- Estimating the loss given default of global events, the proposed methodology provides a sharp yet more conservative estimates when comparing with those provided by the regulator standard.

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ABSTRACT

Incorporating statistical multiple comparisons techniques with credit risk measurement, a new methodology is proposed to construct exact confidence sets and exact confidence bands for a beta distribution. This involves simultaneous inference on the two parameters of the beta distribution, based upon the inversion of Kolmogorov tests. Some monotonicity properties of the distribution function of the beta distribution are established which enable the derivation of an efficient algorithm for the implementation of the procedure. The methodology has important applications to financial risk management. Specifically, the analysis of loss given default (LGD) data are often modeled with a beta distribution. This new approach properly addresses model risk caused by inadequate sample sizes of LGD data, and can be used in conjunction with the standard recommendations provided by regulators to provide enhanced and more informative analyses.

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1. Introduction

Practitioners in risk management necessarily employ a wide array of statistical estimation techniques in their day-to-day activities. However, statistical inferences which quantify the reliability of an adopted statistical model based upon limited data have long been downplayed. For example, a 99% value at risk is regularly computed and shown on a risk management report, even though its confidence interval has generally been ignored. With current advances in multiple comparisons techniques, risk managers should now become equipped with novel statistical inference tools that enable them to easily incorporate statistical inferences into their standard risk management procedures. In this article, we introduce these ideas through multiple comparisons on the beta distribution model with applications to credit risk measurement.

The beta distribution is an important probability distribution whose range is defined on the interval of real numbers between 0 and 1. It may be employed to describe the probabilistic behavior of system responses as a percentage, or to describe the rate of occurrence of an event, for example. Beta distributions also arise in measurements resulting from basic stochastic processes and order statistics. In addition, the beta distribution is the conjugate prior of the binomial likelihood (see, for example, Feller, 1971; Johnson et al., 1995).

The probability density function of the *standard beta distribution* is

$$\frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}$$

where $0 < x < 1$ and $B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx$ is the beta function, which depends upon two positive shape parameters a and b. Therefore, inferences on this distribution require simultaneous inferences of these two parameters, which is the objective of this paper.

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Recent advances in the construction of simultaneous confidence bands for distribution functions have focused on developing exact bands (Frey, 2008; Kiatsupaibul and Hayter, 2015) rather than using the traditional asymptotic bands (Cheng and Iles, 1983; Bickel and Freedman, 1981; Bickel and Krieger, 1989). Parametric exact confidence bands for distributions with multiple parameters have been developed based on nonparametric statistics, such as Kolmogorov statistics. The Weibull distribution (Hayter and Kiatsupaibul, 2013) and the gamma distribution (Hayter and Kiatsupaibul, 2014) are two examples. Exact inferences are possible from the principle of inverse hypothesis testing to construct a confidence set for the parameters at a specified confidence level $1-\alpha$. Confidence bands for the distribution function then follow readily from a mapping of the confidence set into the distribution function space.

This paper shows how exact confidence sets and exact confidence bands can be constructed for the beta distribution based on this methodology. As part of this process, a monotonicity property of the beta distribution function with respect to its parameters is established. This property proves useful in the efficient construction of the confidence set for the parameters, and consequently for the construction of the confidence bands for the distribution.

Confidence bands for distribution functions have important applications to risk management. For example, when a Weibull distribution is adopted as a parametric model for a mortality distribution of interest to an insurer, its confidence band (Hayter and Kiatsupaibul, 2013) can be employed to measure the risk of the portfolio of the life insurance products. In addition, when the arrival times of abnormal internet connections are modeled by a gamma distribution, its confidence band (Hayter and Kiatsupaibul, 2014) provides information about the risk of the internet security system.

Furthermore, in the area of credit risk management the beta distribution is recommended by some standards as a model for the Loss Given Default (LGD). The LGD is a crucial factor in calculating the loss in an event of a credit default, along with the default rate and the exposure at default (see, for example, Gupton et al., 1997; Duffie and Singleton, 2003; Altman, 2008; Frontczak and Rostek, 2015; Wei and Yuan, 2016). However, since there is uncertainty in fitting the LGD model, financial institutions are recommended by the regulators to perform stress tests, and a particularly rigorous way of stress testing the LGD model is to use the confidence band of the model distribution function. Therefore, an important application of the confidence band methodology proposed in this paper can be found in this process of credit risk management, and an example is provided in this paper.

This paper is organized as follows. In Section 2 the proposed methodology is described by first deriving a monotonicity property of the distribution function as functions of its parameters. An algorithm to construct the exact confidence set and confidence band for the beta distribution is then provided. In Section 3 examples of the confidence band construction are given for both simulated data sets and a real data set. In the real data example it is shown how to construct a confidence band for the LGD distribution and a discussion is provided of its application in credit risk management. Finally, a summary is provided in Section 4.

2. Methodology

In this section the proposed methodology for the construction of confidence sets and confidence bands for a beta distribution is provided. The theoretical development is discussed in Section 2.1, and algorithms are outlined in Section 2.2.

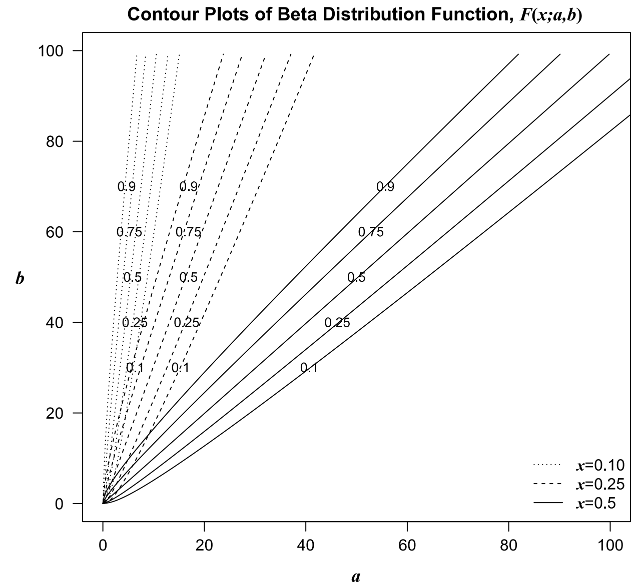

Fig. 1. Some contours of the distribution function $F(x;\,a,\,b)$.

2.1. Theoretical development

Consider a vector of n independent identically distributed random variables $\boldsymbol{X} = (X_1,\ldots,X_n)$ having the beta distribution function $F(x;\,a,\,b)$, with positive shape parameters a and b, given by

$$F(x;\,a,\,b) = \frac{\int_0^x f(t;\,a,\,b)\,dt}{\int_0^1 f(t;\,a,\,b)\,dt} \tag{1}$$

where

$$f(x;\,a,\,b) = x^{a-1}(1-x)^{b-1}.$$

Examples of the distribution function contours at various x on the parameter (a, b) plane are shown in Fig. 1, and notice that $F(x;\,a,\,b) = 1 - F(1-x;\,b,\,a)$. The expectation of this distribution is $a/(a+b)$, which remains unchanged if the parameters a and b are both scaled by the same quantity, and it can be seen that the contours are reasonably straight lines extending out from the origin $a = b = 0$.

Consider the acceptance set $A(a, b)$ defined by

$$A(a,b) = \left\{ \boldsymbol{X} : \sup_x |G_{\boldsymbol{X}}(x) - F(x;\,a,\,b)| \le d_{\alpha,n} \right\}, \tag{2}$$

where $G_{\boldsymbol{X}}(x)$ is the empirical cumulative distribution function of a sample of $\boldsymbol{X}$ and $d_{\alpha,n}$ is the Kolmogorov critical point. It follows from Kolmogorov's test that there is a probability of exactly $1-\alpha$ that the observed $\boldsymbol{X}$ will fall within the acceptance set of the true parameter values. Moreover, Eq. (2) can be written as

$$\frac{i}{n} - d_{\alpha,n} \le F(X_{(i)};\,a,\,b) \le \frac{i-1}{n} + d_{\alpha,n}, \quad i = 1,\ldots,n, \tag{3}$$

where $X_{(1)} \le \cdots \le X_{(n)}$ are the ordered values of $X_1,\ldots,X_n$. Consequently, for an observed value $\boldsymbol{X}$, a $100(1-\alpha)\%$ confidence set $K_\alpha(\boldsymbol{X})$ for the beta parameters is the set of pairs (a, b) for which (3) is satisfied.

It should be noted that the confidence set $K_\alpha(\boldsymbol{X})$ may be empty. This occurs when there are no values of a and b that satisfy Eq. (3), and it alerts the experimenter to the fact that the data should not be modeled with a beta distribution. In other words, the confidence set $K_\alpha(\boldsymbol{X})$ is empty if there is no beta distribution consistent with the data according to Kolmogorov's test.

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