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Contagion modeling between the financial and insurance markets with time changed processes

Donatien Hainaut

ISBA, Université Catholique de Louvain, Voie du Roman Pays 20, B-1348 Louvain-la-Neuve, Belgique

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ABSTRACT

This study analyzes the impact of contagion between financial and non-life insurance markets on the asset–liability management policy of an insurance company. The indirect dependence between these markets is modeled by assuming that the assets return and non-life insurance claims are led respectively by time-changed Brownian and jump processes, for which stochastic clocks are integrals of mutually self-exciting processes. This model exhibits delayed co-movements between financial and non-life insurance markets, caused by events like natural disasters, epidemics, or economic recessions.

1. Introduction

Non life insurance claims, by nature, are not correlated to financial markets, excepted in case of events like natural disasters, epidemics, or serious economic recession. For example, in 2003, the severe acute respiratory syndrome (SARS) spread across several countries and affected with a delay the insurance industry in different ways. Some areas of impacted insurance operations are clear—event cancellations coverage, travel insurance and life and health policies. This epidemic also slowed down economic exchanges and indirectly caused turmoil in financial markets. More recently, during the financial crisis of 2008, the number of claims covered by credit insurances surged in US, as underlined in a recent report from the IMF (2016). As last example, we mention climate changes. It is already affecting and will over time significantly affect the incidence of natural conditions such as: tropical cyclones; winter storms; wild-fires; hail storms; lightning strikes; droughts and floods. These events are expected to affect significantly property claims to non-life insurers. In parallel, climate change will have a huge economic and social impact and will lead to financial instability. These observations motivate us to study the influence of a potential contagion between the insurance and financial markets on the asset–liability management policy of insurers.

The literature about the modeling and management of non-life insurance company is vast. The starting point of research in this field is the classical Cramer–Lundberg (1903) risk model, in which the arrival of claims is modeled by a Poisson process. Since then, many extensions have been developed and proposed bounds on the insurer's ruin probability in various frameworks. Later, Björk and Grandell (1988) and Embrechts et al. (1993) introduced a Cox process in the Cramer–Lundberg model, for the modeling of claim arrivals. Albrecher and Asmussen (2006) studied a Cox process with shot noise intensity. Dassios and Zhao (2011, 2012) analyzed the clustering phenomenon of claims, caused by a self-exciting process. Another strand of the literature focuses on the optimization of investment, reinsurance and dividend policies, in a Cramer–Lundberg approach. For example, Browne (1995) showed in a one-dimensional diffusion model that the strategy maximizing the expected exponential utility of terminal wealth also minimizes the ruin probability. Asmussen and Taksar (1997) studied the optimal dividend policy for an insurer. Hipp and Plum (2000) optimized the investment policy of a non life insurer's surplus, in a Brownian setting. Schmidli (2002, 2006) instead of maximizing the utility of the surplus or dividends, adapted the investment and reinsurance strategies to minimize the probability of ruin. Kaluszka (2001, 2004) examined the optimal reinsurance problem under various mean–variance premium principles. Yuen et al. (2015) considered the optimal proportional reinsurance strategy in a risk model with multiple dependent classes of insurance business. And recently, Yin and Yuen (2015) studied the optimal dividend problems for a jump diffusion model with capital injections and proportional transaction costs. Whereas Zheng et al.

E-mail address: donatien.hainaut@gmail.com.

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(2016) investigated the robust optimal portfolio and reinsurance problem for an ambiguity-averse insurer.

This work studies the optimal proportional reinsurance, dividends and asset allocation for a non-life insurer, in presence of a contagion risk between financial and insurance activities. Drawing on the theoretical and empirical background regarding the market time scale, as in Ané and Geman (2000) or Salmon and Tham (2007), we build time-changed dynamics with chronometers that are integrated positive Hawkes processes. This approach, inspired from Hainaut (2016d) introduces a nonlinear dependence between assets and liabilities. Hawkes processes developed by Hawkes (1971a,b) and Hawkes and Oakes (1974), are parsimonious self-exciting point processes for which the intensity jumps in response and reverts to a target level in the absence of event. This dynamics is increasingly used in finance to model the clustering of shocks. Empirical analysis conducted in Aït-Sahalia et al. (2015, 2014) or in Embrechts et al. (2011) emphasizes the importance of this effect in stocks or CDS markets. They also underline that clustering is not characterized by a single jump but by the amplification of this movement that takes place over days. Recently, Hainaut (2016a,b) detects self-excitation in interest rate markets. And the paper of (Hainaut, 2016c) analyzes the impact of the clustering of jumps on prices and risk of variable annuities. In this work, Hawkes processes determine the pace of market clocks.

This research contributes to the literature in several directions. Firstly, it is an elegant method to introduce dependence between a geometric Brownian motion and a risk process. In this framework, we find the main features of clocks: means, variances and their joint moment generating function (mgf).

Secondly, we show that the linear dependence between log-prices and claims is proportional to the risk premium of stocks and to the insurer's average profit. In particular, when the insurer does not charge any fee above the pure premium, the correlation is null despite an evident dependence by construction. When the insurer's margin is positive, the linear correlation is induced by incomes from the insurance activity, reinvested in the financial market. From an economic point of view, the linear dependence between insurance and financial markets in a time-changed model find its origin in the existence of a risk premium in both segments.

Thirdly, we prove that the insurer's ruin probability is still below the Cramer–Lundberg bound, if the surplus is not invested in the stocks market. Fourthly, we determine the optimal investment, reinsurance and dividend policies that maximize the exponential utility drawn from dividends and terminal surplus. Surprisingly, Optimal reinsurance and investment strategies are independent from markets clocks. Whereas, the optimal dividend is a linear function of the wealth and of intensities of chronometers. Finally, we compare with optimal strategies when the claim process is approached by a Brownian motion.

2. Stochastic clocks of financial and insurance markets

Papers of Ané and Geman (2000) and Salmon and Tham (2007) provide pieces of evidence that the time scale of financial markets is not chronological but rather driven by traded volumes. Starting from this observation, we respectively model financial returns and insurance claims by a Brownian motion and a jump process, observed on distinct random scales of time. This approach allows us to replicate clustering of shocks observed in financial and in insurance markets. It also introduces contagion and random correlation between assets and liabilities. The chronometers measuring the time scales of financial and insurance markets are respectively denoted by τ_t^S and τ_t^L. They are positive increasing processes defined as the integrals of two processes $\left(\lambda_t^S\right)_t$ and $\left(\lambda_t^L\right)_t$ on a probability space Ω, endowed with a probability measure P and a natural filtration denoted by $(\mathcal{G}_t)_t$:

$$\tau_t^S := \int_0^t \lambda_s^S ds \tag{1}$$

$$\tau_t^L := \int_0^t \lambda_s^L ds.$$

By construction, the sample paths of random clocks are continuous and $d\tau_t^S = \lambda_t^S dt$, $d\tau_t^L = \lambda_t^L dt$. λ_t^S and λ_t^L are non-homogeneous processes that may be interpreted as the frequencies at which economic or actuarial information flows. Their dynamics is ruled by two auxiliary jump processes $\left(Z_t^S\right)_t$ and $\left(Z_t^L\right)_t$ such that $Z_t^S = \sum_{k=1}^{N_t^S} J_k^S$ and $Z_t^L = \sum_{k=1}^{N_t^L} J_k^L$. Where $\left(N_t^S\right)_t$ and $\left(N_t^L\right)_t$ are point processes with random jumps J_k^S and J_k^L. The intensities of jump arrivals are assumed equal to the frequencies of information flows: λ_t^S and λ_t^L.

For the sake of simplicity, jumps are exponential random variables with densities $\nu^S(z) = \rho_S e^{-\rho_S z} 1_{\{z\geq 0\}}$ and $\nu^L(z) = \rho_L e^{-\rho_L z} 1_{\{z\geq 0\}}$ where ρ_S and ρ_L are positive and constant. The average sizes of jumps are equal to $\mu_S = \frac{1}{\rho_S}$ and $\mu_L = \frac{1}{\rho_L}$. Whereas the moment generating functions of jumps are respectively $\phi^{J_S}(\omega) := \mathbb{E}\left(e^{\omega J^S}\right)$ and $\phi^{J_L}(\omega) := \mathbb{E}\left(e^{\omega J^L}\right)$. We assume that the couple of intensities λ_t^S and λ_t^L obeys to the next dynamics:

$$\begin{pmatrix} d\lambda_t^S \\ d\lambda_t^L \end{pmatrix} = \begin{pmatrix} \alpha_S\left(\theta_S - \lambda_t^S\right) \\ \alpha_L\left(\theta_L - \lambda_t^L\right) \end{pmatrix} dt + \underbrace{\begin{pmatrix} \eta_{SS} & \eta_{SL} \\ \eta_{LS} & \eta_{LL} \end{pmatrix}}_{\Xi} \begin{pmatrix} dZ_t^S \\ dZ_t^L \end{pmatrix}. \tag{2}$$

These processes revert respectively at speeds α_S and α_L toward θ_S or θ_L. The parameters η_{LS}, η_{SS}, η_{SL}, η_{LL} are constant and positive. This last relation underlines the main features of our approach: contagion, mutual and self-excitation. Indeed, when the clock of the financial (resp. insurance) market speeds up due to a jump of Z_t^S (resp. Z_t^L), the chronometer of the insurance (resp. financial) market accelerates proportionally. This also raises the volatility as longer periods, measured on the market time scale, are observed on the same invariable chronological time scale. Another consequence of a jump is an instantaneous increase in the probability of observing a new financial or actuarial shock as λ_t^S and λ_t^L are the intensities of point processes Z_t^S and Z_t^L. We check by direct differentiation that intensities are the sum of a deterministic function and of two jump processes,

$$\begin{aligned} \lambda_t^S &= \theta_S + e^{-\alpha_S t}\left(\lambda_0^S - \theta_S\right) + \eta_{SS}\int_0^t e^{-\alpha_S(t-s)} dZ_s^S \\ &\quad + \eta_{SL}\int_0^t e^{-\alpha_S(t-s)} dZ_s^L, \\ \lambda_t^L &= \theta_L + e^{-\alpha_L t}\left(\lambda_0^L - \theta_L\right) + \eta_{LS}\int_0^t e^{-\alpha_L(t-s)} dZ_s^S \\ &\quad + \eta_{LL}\int_0^t e^{-\alpha_L(t-s)} dZ_s^L. \end{aligned} \tag{3}$$

These expressions are useful to find closed form expressions of expected intensities, from which we will infer the conditions guaranteeing the stability of jump processes.

Proposition 2.1. *The expectations of λ_t^S and λ_t^L are equal to*

$$\begin{pmatrix} m^S(t) \\ m_0^S(t) \end{pmatrix} := \begin{pmatrix} \mathbb{E}\left(\lambda_t^S | \mathcal{G}_0\right) \\ \mathbb{E}\left(\lambda_t^L | \mathcal{G}_0\right) \end{pmatrix}$$