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Review

Nonparametric estimation of the claim amount in the strong stability analysis of the classical risk model

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ABSTRACT

This paper presents an extension of the strong stability analysis in risk models using nonparametric kernel density estimation for the claim amounts. First, we detail the application of the strong stability method in risk models realized by V. Kalashnikov in 2000. In particular, we investigate the conditions and the approximation error of the real model, in which the probability distribution of the claim amounts is not known, by the classical risk model with exponentially distributed claim sizes. Using the nonparametric approach, we propose different kernel estimators for the density of claim amounts in the real model. A simulation study is performed to numerically compare between the approximation errors (stability bounds) obtained using the different proposed kernel densities.

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Contents

1. Introduction

In ruin theory, stochastic processes are used to model the surplus of an insurance company and to evaluate its ruin probability, i.e., the probability that the total amount of claims exceeds its reserve. This characteristic is a much studied risk measure in the literature. In general, this measure in finite and in infinite time is very difficult or even impossible to evaluate explicitly. Thus, different approximation methods have been proposed to estimate this characteristic (see Asmussen and Albrecher, 2010; Grandell, 1990).

We consider throughout this paper the two risk reserve processes $\{S(t), t \geq 0\}$ and $\{S'(t), t \geq 0\}$ which are given by:

$$S(t) = u + ct - \sum_{i=1}^{N(t)} Z_i, \quad t \geq 0, \tag{1}$$

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$$S'(t) = u + ct - \sum_{i=1}^{N(t)} Z'_i, \quad t \geq 0, \tag{2}$$

where $u \geq 0$ is the initial reserve, $c > 0$ represents the premium rate and $\{N(t), t \geq 0\}$ is a Poisson process with parameter λ. The independent and identically distributed random variables of claim amount $\{Z_i\}_{i\in\mathbb{N}^*}$ and $\{Z'_i\}_{i\in\mathbb{N}^*}$ have distinct distribution functions F and F'.

The question of stability in actuarial risk theory naturally arises for two principal reasons. First, the parameters that govern the risk model are obtained using statistical methods. Second, the ruin probability:

$$\Psi(u) = \mathbb{P}(\exists t \geq 0 | S(t) < 0), \quad \forall u \geq 0, \tag{3}$$

cannot be found explicitly. Hence, it is necessary to obtain explicit stability bounds. The strong stability method, which was developed by Aissani and Kartashov (1983), makes it possible to clarify the conditions for which the ruin probability of the complex risk model (real model) defined by the process (1) can be approximated by the corresponding ruin probability in the simple risk model (ideal model) defined by the process (2). In other words, the model defined by the process (2) may be used as a good approximation or idealization of the real model defined by the process (1).

With a certain norm $\|\cdot\|_v$, Kalashnikov (2000) presented a new stability bound for the ruin probability which has the following form:

$$\|\Psi - \Psi'\|_v \leq \Pi(\|F - F'\|_v, c, \lambda), \tag{4}$$

where Π is a function continuous at 0 with $\Pi(0) = 0$.

In this sense, further work was done for other models: the risk model with investment (Rusaityte, 2001), semi-Markov risk models (Enikeeva et al., 2001) and the two-dimensional classical risk model (Benouaret and Aissani, 2010).

The strong stability analysis is part of the robustness theory, i.e., when we do not know exact values of the model parameters (inputs), it is natural to measure the impact of a small perturbation of the model on the outputs. The influence function, which was used by Marceau and Rioux (2001), Loisel et al. (2008) in the sensitivity and robustness analysis of ruin probabilities, is one of the tools to measure this impact. However, strong stability is another tool to measure the deviation between the ruin probabilities. In contrast to the influence function, this technique based on the disturbance of a linear operator permits us to investigate the ergodicity and the stability of the stationary and non-stationary characteristics of Markov chains (see Aissani and Kartashov, 1984).

There is an alternative method for computing the bounds on the perturbations of Markov chains closely related to the strong stability approach which is the series expansion approach for Markov chains (see Hamoudi et al., 2014). In contrast to the strong stability method, the series expansion approach requires numerical computation of the deviation matrix, which limits the approach to Markov chains with a finite state space (see Heidergott et al., 2010,b).

For a theoretical study, different probability laws can be used to model the amount of claims. In practice, the determination of these probability distributions requires the use of functional estimation techniques (see Bareche and Aissani, 2008, 2010; Zhang et al., 2014). Our contribution in this work is to use the nonparametric estimation of the claim amounts in the strong stability analysis of the ruin probabilities. Assume that the law of the claim amounts is exponential in the ideal risk model described by the process $S'(t)$ and the law of the claim amounts is general in the real risk model described by the process $S(t)$. We clarify, using the strong stability method, the conditions for approximating ruin probabilities Ψ by Ψ' and we estimate the error of this approximation given in bound (4).

This paper is organized as follows: in Section 2, we give the basics of strong stability method applied to the classical risk models. In Section 3, we present some kernels proposed in nonparametric estimation of the density of claim amounts. The main results of this paper are presented and numerically illustrated in Section 4.

2. Preliminaries and position of the problem

In this section, we present some necessary notations, the basic theorems of the strong stability method and the theoretical results obtained by applying this method in the risk models (see Kartashov, 1996; Kalashnikov, 2000; Benouaret and Aissani, 2010).

2.1. The strong stability criteria

We denote by $m\varepsilon$ the space of finite measures on the probabilizable space (E, ε), and we introduce the special family of norms defined by:

$$\|m\|_v = \int_E v(x)|m|(dx), \quad \forall\, m \in m\varepsilon, \tag{5}$$

where v is a measurable function that is bounded below away from zero (not necessarily finite).

This norm induces, in the space $f\varepsilon$ of bounded measurable functions on E, the norm:

$$\|f\|_v = \sup\{|mf|, \|mf\|_v \leq 1\} = \sup_{x\in E}[v(x)]^{-1}|f(x)|, \quad \forall f \in f\varepsilon. \tag{6}$$

The norm of the transition kernel P in the space β is given as follows:

$$\|P\|_v = \sup_{x\in E}\left([v(x)]^{-1}\int_E v(y)|P(x, dy)|\right), \tag{7}$$

where β is the space of linear operators.

Definition 2.1 (*see Aissani and Kartashov, 1983*)**.** The Markov chain X with transition kernel P and stationary distribution π is said to be v-strongly stable with respect to the norm $\|.\|_v$ if $\|P\|_v < \infty$ and each stochastic kernel Q in the neighborhood $\{Q : \|Q - P\|_v < \epsilon\}$ has a unique invariant measure $\pi' = \pi'(Q)$ and $\|\pi - \pi'\|_v \to 0$ as $\|Q - P\|_v \to 0$.

The following theorem was proved by Kartashov (1996) and was applied in a risk model with one line of business by Kalashnikov (2000).

Theorem 2.1. *Let v be a fixed weight function. Consider a Markov chain with transition kernel P, such as $\|P\|_v < \infty$, and that has a unique stationary distribution π. Additionally, suppose that there is a non-negative function h and a probability measure α such that P can be decomposed as follows:*

$$P(u, \cdot) = T(u, \cdot) + h(u)\, \alpha(\cdot), \tag{8}$$

where

$$\|\pi\|_h > 0, \qquad \|\alpha\|_h > 0, \tag{9}$$

and

$$\|T\|_v \leq \rho < 1. \tag{10}$$

Then, all Markov chains with transition kernel P' that satisfies:

$$\Delta = \|P - P'\|_v < \Delta_0 \equiv \frac{(1-\rho)^2}{1 - \rho + \rho\, \|\alpha\|_v}, \tag{11}$$

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