Insurance: Mathematics and Economics 74 (2017) 135–146

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Optimal periodic dividend and capital injection problem for spectrally positive Lévy processes

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ARTICLE INFO

Article history:
Received May 2016
Received in revised form
March 2017
Accepted 22 March 2017
Available online 30 March 2017

JEL classification:
C44
C61
G32
G35

Keywords:
Periodic dividend
Capital injection
Lévy process
Stochastic control
Scale function

ABSTRACT

In this paper, we investigate an optimal periodic dividend and capital injection problem for spectrally positive Lévy processes. We assume that the periodic dividend strategy has exponential inter-dividend-decision times and continuous monitoring of solvency. Both proportional and fixed transaction costs from capital injection are considered. The objective is to maximize the total value of the expected discounted dividends and the penalized discounted capital injections until the time of ruin. By the fluctuation theory of Lévy processes in Albrecher et al. (2016), the optimal periodic dividend and capital injection strategies are derived. We also find that the optimal return function can be expressed in terms of the scale functions of Lévy processes. Finally, numerical examples are studied to illustrate our results.

1. Introduction

The Lévy risk model with positive jumps (spectrally positive Lévy process), which is also called dual risk model, is proposed to offset continuous expenses by the stochastic and irregular gains. Examples include research-based or commission-based companies. In this context, dividend optimization problems have attracted extensive attention. In Avanzi and Gerber (2008) and Avanzi et al. (2007), the authors studied how the expectation of the discounted dividends until ruin can be calculated in the dual compound Poisson risk model. In Bayraktar et al. (2013), Kyprianou et al. (2012), Yin and Wen (2013) and Zhao et al. (2015), the optimal dividend problems were studied in a general spectrally positive Lévy risk model.

In the above papers, the dividend decisions are made continuously, which usually leads to very irregular dividend payments. However, in practice, the companies that are capable of distributing dividends make dividend decisions on a periodic basis. Albrecher et al. (2011a) studied the random inter-dividend-decision times in the Cramér–Lundberg model, where the ruin cannot occur between dividend payment times. Continuous monitoring of solvency with periodic dividends was considered by Albrecher et al. (2011b) in the Brownian risk model, Avanzi et al. (2013) and Avanzi et al. (2014) in the dual compound Poisson model. Recently, when the inter-dividend-decision times of periodic dividend are exponential, Pérez and Yamazaki (2016) showed the optimality of periodic barrier strategy for spectrally positive Lévy processes.

When dividend payments are maximized, ruin is usually certain. In some cases, it may be profitable to rescue the company by capital injection. This idea goes back to Porteus (1977). Yao et al. (2011) considered an optimal dividend and capital injection problem in the dual compound Poisson risk model. Avanzi et al. (2011) discussed the same problem in the dual compound Poisson model with diffusion. Bayraktar et al. (2013) and Zhao et al. (2015) extended their work to general spectrally positive Lévy processes. In addition, transaction cost, which usually includes two parts:

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http://dx.doi.org/10.1016/j.insmatheco.2017.03.006
0167-6687/

proportional cost and fixed cost, is an important factor in business activities. In Avanzi et al. (2011), the proportional transaction costs from dividend and capital injection were involved into an optimal dividend problem. In Yao et al. (2011) and Zhao et al. (2015), both proportional and fixed costs on capital injection were considered. Fixed costs on dividend were studied in Bayraktar et al. (2014).

In this paper, the optimal periodic dividend and capital injection problem is discussed for spectrally positive Lévy processes. For periodic dividend, we assume that the inter-dividend-decision times are exponential as in Avanzi et al. (2014) and Pérez and Yamazaki (2016), but different methods are used. For capital injection, we include the proportional and fixed transaction costs. We also assume that the ruin may occur even under the rescue of capital injection. Like Zhao et al. (2015), we decompose the optimal problem into two suboptimal problems. By the fluctuation theory of Lévy processes observed at Poisson arrival in Albrecher et al. (2016), we find the optimal strategy and the optimal return function. If the fixed transaction cost from capital injection tends to zero, we obtain the results in Pérez and Yamazaki (2016). When the positive jumps of the Lévy process are hyper-exponential compound Poisson distributed, the first suboptimal problem becomes that in Avanzi et al. (2014). If the dividend decision intensity goes to infinity, and meanwhile the fixed costs on capital injection tend to zero, the two suboptimal problems in this paper reduce to those in Bayraktar et al. (2013). Furthermore, for hyper-exponential compound Poisson positive jumps, our results are consistent with Avanzi et al. (2011).

This paper is organized as follows. Section 2 provides the formulations of the problem. Section 3 discusses the case without capital injection. The case with incorporated capital injection is considered in Section 4. The optimal periodic dividend and capital injection strategies are derived in Section 5. Section 6 gives some numerical examples.

2. Model and optimal control problem

2.1. Spectrally positive Lévy processes

Let $X = \{X_t\}$ be a spectrally positive Lévy process with non-monotone paths on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$, where $\mathbb{F} = \{\mathcal{F}_t\}$ satisfies the usual conditions. The Lévy triplet of X is given by (c, σ, ν), where $c > 0$, $\sigma \geq 0$, and ν is a Lévy measure on $(0, \infty)$ satisfying the integrability condition $\int_0^\infty (1 \wedge x^2)\nu(dx) < \infty$. Let E^x be the conditional expectation given the initial surplus x, and in particular, denote E^0 by E. The Laplace exponent of X is given by

$$\psi(s) = \frac{1}{t}\log E\left[e^{-sX_t}\right] = \frac{\sigma^2}{2}s^2 + cs + \int_0^\infty (e^{-sx} - 1 + sx\mathbf{1}_{\{0<x\leq 1\}})\nu(dx), \tag{2.1}$$

where $\mathbf{1}_A$ is an indicator function of a set A. It is well known that $\psi(s)$ is strictly convex and tends to infinity as s tends to infinity. This allows us to define for $q \geq 0$,

$$\Phi(q) = \sup\{s \geq 0 : \psi(s) = q\}, \tag{2.2}$$

which refers to the largest root of the equation $\psi(s) = q$. Note that the Laplace exponent ψ in (2.1) is known to be zero at the origin, and hence $\Phi(q)$ is strictly positive for $q > 0$. The process X has paths of bounded variation if and only if $\sigma = 0$ and $\int_0^1 x\nu(dx) < \infty$. Correspondingly, the Laplace exponent (2.1) can be written as

$$\psi(s) = c_0 s + \int_0^\infty (e^{-sx} - 1)\nu(dx), \tag{2.3}$$

where $c_0 = c + \int_0^1 x\nu(dx)$. We rule out the case that X has monotone paths, and then $c_0 > 0$ is necessary when X is of bounded variation. The drift of X is given by

$\mu = E[X_1] = -\psi'(0+)$.

It is well known that if $\int_1^\infty y\nu(dy) < \infty$, then $\mu = -c + \int_1^\infty y\nu(dy) < \infty$. In this paper, we assume $\mu < \infty$ to ensure that the optimal problem has a nontrivial solution. For more details on Lévy processes, the reader is referred to Bertoin (1996), Kyprianou (2006) and Kuznetsov et al. (2012).

2.2. Formulations of control problem

We assume that the surplus process of a company is modeled by the Lévy process X, whose Laplace exponent is given by (2.1) and $X_0 = x \geq 0$. A control strategy is composed of two parts: dividend payment and capital injection. We assume that the dividend payments follow a periodic dividend strategy, where the dividend decision times are governed by a $\mathbb{F}$-adapted and independent Poisson process $N = \{N_t\}$ with intensity $\gamma > 0$. Then the dividend payment process $L = \{L_t\}$ is given by

$$L_t = \int_0^t \vartheta_s dN_s = \sum_{k=1}^\infty \vartheta_{T_k}\mathbf{1}_{\{T_k\leq t\}}, \quad t \geq 0,$$

where $\vartheta_t \geq 0$ is the dividend payment at time t, and T_k is the time of kth dividend. The capital injection process $G = \{G_t = \sum_{n=1}^\infty \mathbf{1}_{\{\tau_n\leq t\}}\xi_n\}$ is described by a sequence of increasing stopping times $\{\tau_n, n = 1, 2, \ldots\}$ and a sequence of random variables $\{\xi_n, n = 1, 2, \ldots\}$, which correspond to the timings and the amounts of capital injection, respectively.

Given a control strategy $\pi = (L^\pi; G^\pi) = (\vartheta^\pi_{T_1}, \vartheta^\pi_{T_2}, \ldots; \tau^\pi_1, \tau^\pi_2, \ldots; \xi^\pi_1, \xi^\pi_2, \ldots)$, the dynamics of the resulting surplus process $X^\pi = \{X^\pi_t\}$ can be written as

$$X^\pi_t = X_t - \sum_{k=1}^\infty \vartheta^\pi_{T_k}\mathbf{1}_{\{T_k\leq t\}} + \sum_{n=1}^\infty \xi^\pi_n \mathbf{1}_{\{\tau^\pi_n\leq t\}}, \quad t \geq 0. \tag{2.4}$$

Definition 2.1. A strategy π is said to be admissible iff

(i) $\{L^\pi_t\}_{t\geq 0}$ is an increasing and $\mathbb{F}$-adapted càdlàg process satisfying $L_0 = 0$,
$L^\pi_t = \int_0^t \vartheta^\pi_s dN_s$ and $\Delta L^\pi_{T_j} = \vartheta^\pi_{T_j} \leq X^\pi_{T_j-} + \Delta X_{T_j}$ for $j = 1, 2, \ldots$;
(ii) τ^π_n is a stopping time with respect to $\mathbb{F}$, and $0 \leq \tau^\pi_1 < \tau^\pi_2 < \cdots$ a.s.;
(iii) ξ^π_n is nonnegative and measurable with respect to $\mathcal{F}_{\tau^\pi_n}$, $n = 1, 2, \ldots$;
(iv) $P(\lim_{n\to\infty} \tau^\pi_n \leq a) = 0, \forall a \geq 0$.

Let Π denote the set of admissible strategies. Define the time of ruin by

$T^\pi = \inf\{t \geq 0 : X^\pi_t \leq 0\}$,

with $\inf\emptyset = \infty$ by convention. To incorporate the capital injection, the fixed and proportional transaction costs are considered. When the amount of capital is ξ, we assume that $(\phi - 1)\xi$ with $\phi > 1$ is the proportional cost, and that $K > 0$ is the fixed cost. The time preference of investors is described by the force of interest $\delta > 0$. Given a strategy $\pi \in \Pi$, the performance function is defined by

$$V(x;\pi) = E^x\left[\int_0^{T^\pi} e^{-\delta s}\vartheta^\pi_s dN_s - \sum_{n=1}^\infty e^{-\delta\tau^\pi_n}(K + \phi\xi^\pi_n)\mathbf{1}_{\{\tau^\pi_n\leq T^\pi\}}\right]. \tag{2.5}$$

Our objective is to find the optimal return function or the value function, defined as

$$V(x) = \sup_{\pi\in\Pi} V(x;\pi), \tag{2.6}$$

and the associated optimal strategy $\pi^* \in \Pi$ such that $V(x) = V(x;\pi^*)$.