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Exponential utility maximization for an insurer with time-inconsistent preferences[☆]

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HIGHLIGHTS

- A utility maximization problem is studied with a general discount function.
- Some of the parameters in the model are adapted stochastic processes.
- The methods of multi-person game and of martingale are used.
- A time-consistent equilibrium strategy is got by using BSDE and integral equation.

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ABSTRACT

This paper studies the optimal consumption–investment–reinsurance problem for an insurer with a general discount function and exponential utility function in a non-Markovian model. The appreciation rate and volatility of the stock, the premium rate and volatility of the risk process of the insurer are assumed to be adapted stochastic processes, while the interest rate is assumed to be deterministic. The object is to maximize the utility of intertemporal consumption and terminal wealth. By the method of multi-person differential game, we show that the time-consistent equilibrium strategy and the corresponding equilibrium value function can be characterized by the unique solutions of a BSDE and an integral equation. Under appropriate conditions, we show that this integral equation admits a unique solution. Furthermore, we compare the time-consistent equilibrium strategies with the optimal strategy for exponential discount function, and with the strategies for naive insurers in two special cases.
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1. Introduction

Reinsurance and investment are two effective ways to control and manage risk of an insurer. The optimization of reinsurance and investment has been investigated by many researchers under various criteria. For the maximization of the exponential utility of terminal wealth, see Browne (1995), Yang and Zhang (2005) and Wang et al. (2007), among others. In addition, to keep the businesses running and share profit with the shareholders, the insurer has consumption expenditure that reduces her surplus. This motivates us to study the optimization of consumption, investment and reinsurance for an insurer. Under the criterion of maximizing the exponential utility of intertemporal consumption and terminal wealth, the problem is studied in Peng et al. (2014a,b), among others.

In most of existing papers, it is standard to assume that the insurer has a constant rate of time preference (exponential discounting). Such preferences are time-consistent in that an insurer's preference for rewards at an earlier date over a later date is the same no matter when he/she is asked. However, the assumption of time-consistency has been challenged by some economic literature, see, for example, Thaler (1981), Ainslie (1992) and Loewenstein and Prelec (1992). Experimental evidence shows that economic agents are impatient about choices in short term but are more patient when choosing between long-term alternatives. Considering such behavioral feature, economic decisions may be analyzed using the hyperbolic discounting. For more literature on the time-inconsistent preferences, we refer the reader to a review paper by Frederick et al. (2002), and the references therein.

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With non-exponential discounting, the optimal control problem becomes time-inconsistent, i.e, a strategy that is optimal for the initial time may not be optimal later (see, for example, Ekeland and Pirvu (2008) and Yong (2012)). In other words, the classical dynamic programming principle fails to solve the so-called time-inconsistent control problems. Therefore, how to obtain a time-consistent strategy for time-inconsistent control problems becomes an interesting and challenging problem. Strotz (1955) studies a cake-eating problem within a game theoretic framework where the players are the agent and his/her future selves, and seek a subgame perfect Nash equilibrium point for this game. Strotz's work has been pursued by many others, such as Pollak (1968), Peleg and Yaari (1973), Goldman (1980) and Laibson (1997) among others, mostly in discrete-time models.

In recent years, time inconsistent control problems regain considerable attention in continuous-time settings. The optimal consumption and investment problem with non-constant discount rate is studied by Marín-Solano and Navas (2010) in Merton's model for both naive and sophisticated agents by solving a modified HJB equation. The same problem is also considered by Ekeland and Lazrak (2006) and Ekeland and Pirvu (2008), which give the precise definition of the equilibrium concept in continuous-time model. There are some literature following their definition of equilibrium strategy. In Björk and Murgoci (2010), the time-inconsistent control problem is considered in a general Markovian framework, and an extended HJB equation together with a verification theorem are derived. Considering the hyperbolic discounting, Ekeland et al. (2012) studies the portfolio management problem in Richard's model, and they characterize the equilibrium strategy by an integral equation.

Another approach to the time-inconsistent control problem, i.e., the method of multi-person game, is developed by Yong (2011, 2012). Yong's idea is to partition a given time interval into small subintervals and to consider a decision-making problem at each interval as if it is made by an independent player. Thus the original optimization problem is transformed into a multi-person game, and the concept of optimality is replaced by that of Nash equilibrium. Then, a time-consistent equilibrium strategy and equilibrium value function are obtained by letting the mesh size of the partition go to zero. Through this method, Yong (2011) considers a deterministic time-inconsistent linear–quadratic control problem. Considering a controlled stochastic differential equation with deterministic coefficients, Yong (2012) investigates a time-inconsistent problem with a general cost functional and derives an equilibrium HJB equation. Yong (2013) studies a time-inconsistent linear–quadratic control problems for mean-field stochastic differential equations.

In this paper, we consider the optimal consumption–investment–reinsurance problem for an insurer with a general discount function and exponential utility function. The insurer is allowed to invest in a financial market with multiple assets. In contrast to Ekeland and Pirvu (2008) and Marín-Solano and Navas (2010), we work in a non-Markovian model. More specifically, the appreciation rate and volatility of the stock, the premium rate and volatility of the risk process of the insurer are assumed to be adapted stochastic processes, while the interest rate is assumed to be deterministic.[1] The object is to maximize the utility of intertemporal consumption and terminal wealth. The model is similar to the one adopted in Peng et al. (2014b), where the insurance risk and financial risk are independent. Since the insurer's surplus may be affected by the performance of the financial market, we consider the dependence between the insurance risk and financial risk.

[1] This is a technical assumption which is needed to get an explicit solution to our problem, see Section 6.

To obtain a time-consistent strategy, we adopt the method proposed by Yong (2011, 2012). First, we introduce a multi-person differential game. By the martingale method (see Hu et al., 2005; Cheridito and Hu, 2011), the optimal strategy of each player is characterized by the unique solution of an ordinary differential equation (ODE, for short). Then, letting the mesh size of the partition go to zero, we show that the time-consistent equilibrium strategy and the corresponding equilibrium value function can be characterized by the unique solutions to a backward stochastic differential equation (BSDE, for short) and an integral equation. The existence and uniqueness of the solution to this integral equation are also investigated. In the cases with mixture of exponential discount functions and hyperbolic discount function, we compare the time-consistent equilibrium strategies with the optimal strategy for exponential discount function, and with the strategies for naive insurers. The results show that at any time, given the same wealth and discount rate, the insurer with non-exponential discount function consumes less than the one with exponential discount function, and the time-consistent equilibrium consumption strategy is smaller than the naive consumption strategy. Furthermore, it shows that the time-consistent equilibrium strategy outperforms the naive strategy as the former gives greater value function.

There are at least two motivations to choose the method of multi-person game. First, it allows us to use standard stochastic control tools, such as BSDEs and martingale approach. This is especially important for studying the problem in this paper, due to the non-Markovian model and an intrinsic constraint on the reinsurance strategy.[2] Second, it enables us to extend the results obtained in this paper to more general cases, where the definition of equilibrium strategy proposed by Ekeland and Lazrak (2006) and Ekeland and Pirvu (2008) may not be applied. See Remark 2.1 for more details.

The remainder of this paper is organized as follows. Section 2 introduces the model. In Section 3, we study a multi-person differential game. Section 4 gives a time-consistent equilibrium strategy and time-consistent equilibrium value function for the original problem. Section 5 presents some comparisons with the optimal strategy for exponential discount function, and with the strategies for naive insurers in two special cases. Section 6 concludes the paper. Some proofs and technical results are collected in the appendices.

2. The model

Let $T > 0$ be a fixed time horizon, $W(\cdot) \equiv (W_1(\cdot), \ldots, W_d(\cdot))^\top$ a d-dimensional standard Brownian motion and $\tilde{W}(\cdot) \equiv \left(\tilde{W}_1(\cdot), \ldots, \tilde{W}_{\tilde{d}}(\cdot)\right)^\top$ a $\tilde{d}$-dimensional standard Brownian motion defined on the filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\le t\le T}, P\right)$. Here, the filtration $\{\mathcal{F}_t\}_{0\le t\le T}$ is the augmentation under P of $\mathcal{F}_t^{W,\tilde{W}} := \sigma\left(W(s), \tilde{W}(s), 0 \le s \le t\right), t \in [0, T]$. We assume that $W(\cdot)$ and $\tilde{W}(\cdot)$ are independent of each other and denote by $\mathbf{W}(\cdot)$ the $d + \tilde{d}$-dimensional standard Brownian motion $\binom{W(s)}{\tilde{W}(s)}$.

Consider an insurer with a surplus process (without any control) that follows the dynamics

$$\begin{cases} dR(s) = \alpha(s)ds + \beta_1(s)dW(s) + \beta_2(s)d\tilde{W}(s), \\ \qquad s \in [0, T], \\ R(0) = r_0, \end{cases} \tag{2.1}$$

[2] Similar to Cheridito and Hu (2011) and Peng et al. (2014b), we may also consider a general constraint on the investment strategy. In this paper, for simplicity and focusing on the time-inconsistency, we only consider the constraint on the reinsurance strategy.

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