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Comparing risks with reference points: A stochastic dominance approach

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ABSTRACT

This paper develops a stochastic dominance rule for the reference-dependent utility theory proposed by Kőszegi and Rabin (2007). The new ordering captures the effects of loss aversion and can be used as a semi-parametric approach in the comparison of risks with reference points. It is analytically amenable and possesses a variety of intuitively appealing properties, including the abilities to identify both "increase in risk" and "increase in downside risk", to resolve the Allais-type anomalies, to capture the violation of translational invariance and scaling invariance, and to accommodate the endowment effect for risk. The generalization to third-order dominance reveals that loss aversion can either reinforce or weaken prudence, depending on the location of the reference point. Potential applications of the new ordering in financial contexts are briefly discussed.

1. Introduction

Reference points such as benchmarks or targets manifest themselves pervasively in portfolio management. Regardless of being individual or institutional, investors commonly have a benchmark to follow or a target to beat. Reference points usually exert substantial influence on investors' risk appetites, which further motivate their strategies (Dittmann et al., 2010).

In the financial literature, the evaluation of risks with reference points is often performed using risk measures such as fixed-target lower-partial moments, downside betas, Value-at-Risk and ExpectedShortfall.[1] These measures are designed primarily on the basis of computational considerations and lack a choice-theoretic foundation. As response to this shortcoming, the literature has suggested a non-parametric approach for evaluating risks based on stochastic dominance rules. These rules are rooted in utility theory and rank prospects with minimal assumptions about investors' risk attitudes (Levy, 1992). However, to the best of our knowledge, with few exceptions (Baucells and Heukamp, 2006), researchers have not yet developed stochastic dominance rules that are suited for comparing risks with explicit reference points. The behavioral stochastic dominance rules such as "prospect stochastic dominance" and "Markowitz stochastic dominance" studied previously mainly focus on exploring the concavity and convexity of the value function, and thereby cannot provide guidance on how to properly mitigate downside risk.

This paper represents a first effort to tailor a stochastic dominance rule suited for comparing risks with reference points. In modeling agents' preferences with reference points, the reference-dependent utility theory developed by Kőszegi and Rabin (2006, 2007) is a popular candidate. To invoke this theory, one needs a von Neumann–Morgenstern utility function to describe the intrinsic taste for outcomes, a parameter to capture the magnitude of loss aversion, and a parameter to measure the weight of gain–loss utility. In this paper, we develop a stochastic dominance rule for Kőszegi and Rabin's theory to reduce the parametrization. To invoke our stochastic dominance rule, one does not need to know exactly the form of the von Neumann–Morgenstern utility function or the values of the two parameters. What is needed is just a

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1 Lower-partial moment models were first introduced by Markowitz (1959). Fishburn (1977) and Price et al. (1982) further developed more general forms of downside risk measures based on fixed-target lower-partial moments. Downside betas were first advocated by Bawa and Lindenberg (1977) and have been adopted by many authors such as Ang et al. (2006). Value-at-Risk and Expected Shortfall are rooted in the safety-first criterion proposed by Roy (1952) and use a reference percentile.

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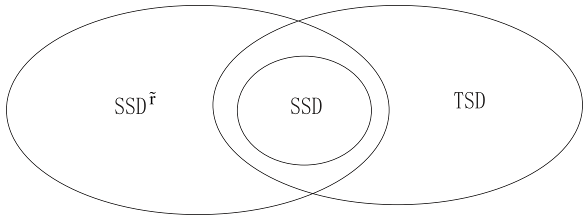

Fig. 1. The inclusion relationship between $SSD^{\tilde{r}}$, SSD and TSD. The relationship $SSD \subset SSD^{\tilde{r}}$ ($SSD \subset TSD$) denotes that for any two random variables, $\tilde{w}_1 SSD \tilde{w}_2$ implies $\tilde{w}_1 SSD^{\tilde{r}} \tilde{w}_2$ ($\tilde{w}_1 TSD \tilde{w}_2$), but the converse is not true. The relationship $(SSD^{\tilde{r}} \cap TSD) \setminus SSD \neq \emptyset$ states that there are pairs of random variables that cannot be ranked by SSD but can be ranked by TSD and $SSD^{\tilde{r}}$ in the same order.

judgment about whether the von Neumann–Morgenstern utility is increasing or concave and whether the two parameters are bigger than some pre-specified lower bounds. As we have to specify the reference point and the lower bounds of the parameters in the piecewise linear value function when employing our dominance rule, we regard it as a semi-parametric approach for comparing risks.

In developing the stochastic dominance rule, we follow Kőszegi and Rabin (2006, 2007) to abstract from the S-shaped nature of the value function by assuming a piecewise linear value function. Piecewise linearity captures risk aversion over gains but precludes risk-lovingness over losses, which is consistent with portfolio mangers' disaster avoidance motive, making our dominance rule suitable for use in practice.[2] We use three specifications of the reference point: (i) exogenous and deterministic; (ii) exogenous and stochastic; and (iii) endogenous (stochastic or not). Among the them, (i) provides a basis where the intuitions underlying our stochastic dominance rule will be elaborated. Most of our discussion is confined to the second-order dominance, while a generalization to higher orders will also be explored. Our new dominance rule proves to be a convenient tool for analytically characterizing the effects of reference point and loss aversion on the risk-taking behavior. When the reference point is exogenous (whether deterministic or stochastic), our new ordering exhibits four features that distinguish it from the traditional dominance rules.

First, compared with the traditional second-order stochastic dominance ("*SSD*", henceforth), the new ordering admitting a reference point $\tilde{r}$, denoted by $SSD^{\tilde{r}}$, shows higher aversion towards the risk spread: in the $SSD^{\tilde{r}}$ ordering, not only all mean-preserving spreads are disliked, but also certain kinds of mean-increasing spreads satisfying the condition specified in Proposition 4 will be disliked. For a spread to be preferred according to $SSD^{\tilde{r}}$, the average increase in return must be high enough to compensate for the downside risk.

Second, the inclusion relationship between $SSD^{\tilde{r}}$, SSD and third-order stochastic dominance (TSD) is found to satisfy Fig. 1. On the one hand, $SSD^{\tilde{r}}$ contains SSD as a sub-ordering and can rank certain risk pairs in the same order as TSD does when risk pairs satisfy the condition specified in Proposition 5. On the other hand, $SSD^{\tilde{r}}$ is definitely a new ordering that is neither sufficient nor necessary for TSD. Therefore, SSD^{r} offers an effective complement to SSD and TSD in identifying the "increase in risk" and the "increase in downside risk" ("more skewed to the left").

Third, since investors' sense of loss is influenced by the location and scale of the risk, $SSD^{\tilde{r}}$ is not invariant under either translations or nonnegative scaling of underlying risks. In particular, as we will see in Proposition 6, the original undominated component can become dominated after an upward scaling of a pair of risks, capturing that investors exhibit larger risk aversion in the face of risks with larger scale.

Fourth, $SSD^{\tilde{r}}$ offers an analytical characterization of the endowment effect for risk. We show in Proposition 8 that the investor can become less averse to the spread of risk when the reference point becomes more dispersed. In other words, the investor is less risk averse if she expects to face greater risks. This effect was firstly formalized by Kőszegi and Rabin (2007) under the assumption of linear intrinsic utility function (see their Proposition 1 on pg. 1053) and has found an experimental support in Sprenger (2015). Our new ordering generalizes the analysis to arbitrary concave utility functions.

Our stochastic dominance rule can easily accommodate endogenous reference points. When reference points are endogenously formed by expectations as in Kőszegi and Rabin (2006, 2007), we show in Proposition 9 that our ordering yields a new resolution of the Allais-type anomalies that includes both the common consequence effect and the common ratio effect. The essence of the resolution is that anticipating more risky lotteries drives investors to become less risk-averse by making their reference points more dispersed.

Our stochastic dominance rule can also be generalized to explore the effects of reference points on higher-order preferences. Although the effects of reference points on the second-order preference (risk aversion) are well studied, little is known on how reference points change higher-order preferences. Maier and Rüger (2012) find that if the reference point is endogenously formed by expectations, individuals exhibit even- but never uneven-order risk attitudes. Complementary to this result, our third-order stochastic dominance rule provided in Proposition 10 shows that when the reference point is exogenous, loss aversion can either reinforce or weaken prudence, depending on the location of the reference point.

We contribute to the literature on risk theory by offering a new method rooted in utility theory for comparing risks with reference points. It establishes robust predictions based on limited information about the utility function, and can be used in experiments as a guide to design pairs of prospects to examine theories with reference-dependence structure (Baucells and Heukamp, 2006). It also adds to the literature on behavioral portfolio (Shefrin and Statman, 2000; Berkelaar et al., 2004; Jarrow and Zhao, 2006; De Giorgi and Post, 2011; He and Zhou, 2011) by providing a new approach in identifying downside risk. Mathematically, $SSD^{\tilde{r}}$ is a simple extension of that of SSD, as it just introduces a stepwise weighting function to the integrand. This makes the existing statistical approaches for testing SSD such as Davidson and Duclos (2000) readily extendable for testing $SSD^{\tilde{r}}$.

The remainder of this paper is structured as follows. Sections 2 and 3 are devoted to the baseline case where the reference point is non-stochastic and exogenous. We concentrate on second-order preference in these two sections. Sections 4–6 are three important extensions, generalizing the analysis to accommodate stochastic reference points, endogenous reference points, and higher-order preferences, respectively. Section 7 concludes this paper with a discussion of potential applications. All proofs are relegated to Appendix.

2. Definition and characterizations of SSD^{r}

We begin with the baseline case where the reference point is constant and exogenous, denoted by r. We use $\tilde{w}$ to denote a random variable and w its realization. When the random variable $\tilde{w}$ is discrete, we use

$$\tilde{w} = (w_1,\ p_1; \cdots ;\ w_n,\ p_n)$$

[2] Convexity for losses found in experiments is not very pronounced. See Abdellaoui et al. (2005).