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A neural network approach to efficient valuation of large portfolios of variable annuities

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ABSTRACT

Managing and hedging the risks associated with Variable Annuity (VA) products require intraday valuation of key risk metrics for these products. The complex structure of VA products and computational complexity of their accurate evaluation have compelled insurance companies to adopt Monte Carlo (MC) simulations to value their large portfolios of VA products. Because the MC simulations are computationally demanding, especially for intraday valuations, insurance companies need more efficient valuation techniques. Recently, a framework based on traditional spatial interpolation techniques has been proposed that can significantly decrease the computational complexity of MC simulation (Gan and Lin, 2015). However, traditional interpolation techniques require the definition of a distance function that can significantly impact their accuracy. Moreover, none of the traditional spatial interpolation techniques provide all of the key properties of accuracy, efficiency, and granularity (Hejazi et al., 2015). In this paper, we present a neural network approach for the spatial interpolation framework that affords an efficient way to find an effective distance function. The proposed approach is accurate, efficient, and provides an accurate granular view of the input portfolio. Our numerical experiments illustrate the superiority of the performance of the proposed neural network approach compared to the traditional spatial interpolation schemes.

1. Introduction

A Variable Annuity (VA), also known as a segregated fund in Canada, is a type of mutual fund that comes with insurance features and guarantees. VAs allow policyholders to invest in financial markets by making payment(s) into a predefined set of sub-accounts set up by insurance companies and enjoy tax-sheltered growth on their investment. The insurer, later, returns these investments through a lump-sum payment or a series of contractually agreed upon payments. An attractive feature of VA products is the embedded guarantees that protect the investment of policyholders from downside market fluctuations in a bear market and mortality risks (TGA, 2013; Chi and Lin, 2012). For a detailed description of VA products and different types of guarantees offered in these products, see our earlier paper (Hejazi et al., 2015) and the references therein.

The innovative structure of embedded guarantees has made VA products a huge success. Major insurance companies, especially in the past decade, have sold trillions of dollars worth of these products (IRI, 2011), and have built up large portfolios of VA contracts, each with hundreds of thousands of contracts. The embedded guarantees of VA contracts in these portfolios expose insurance companies to a substantial amount of risk, such as market risk and behavioral risk. After the market crash of 2008 that wiped out several big insurance companies, the surviving insurance companies started major risk management initiatives to dynamically hedge (Hardy, 2003) their exposures.

An integral part of the aforementioned hedging programs is intraday evaluation of VA products to find the Greeks (Hull, 2006) for the portfolios of VA products so that effective hedging positions can be set up. Most of the academic methodologies for valuation of VA contracts are tailored to a specific type of VA contract (Milevsky and Salisbury, 2006; Chen and Forsyth, 2008; Chen et al., 2008; Dai et al., 2008; Ulm, 2006; Huang and Forsyth, 2011; Belanger et al., 2009) and/or are computationally too expensive to scale to large portfolios of VA contracts (Azimzadeh and Forsyth, 2015; Moenig and Bauer, 2011; Boyle and Tian, 2008). Hence, in practice, insurance companies have relied on nested MC simulations to find the Greeks of VA portfolios (Reynolds and Man, 2008). Nested MC simulations, as shown in Fig. 1, consist of outer loop scenarios which span the space of key market variables and inner loop

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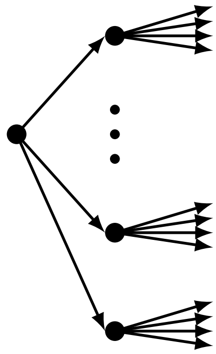

Fig. 1. Pictorial graph of nested MC simulations.

scenarios consisting of a collection of risk-neutral paths that are used to project the liabilities of VA contracts (Fox, 2013). Although MC simulations are computationally less expensive than the academic methodologies, the amount of computation is still significant and does not scale well to large portfolios of VA contracts. Because of this, insurance companies are actively looking for ways to reduce the number of required MC simulations to find the Greeks for a large portfolio of VA contracts.

As we discuss in Section 2, a framework based on spatial interpolation (Burrough et al., 1998) has been successful in ameliorating the computational load of MC simulations by reducing the number of VA contracts that go through nested MC simulation. However, as we discussed in Hejazi et al. (2015), the proposed spatial interpolation framework requires an effective choice of distance function and a sample of VA contracts from the space in which the input portfolio is defined to achieve an acceptable accuracy level. The appropriate choice of the distance function for the given input portfolio in the proposed framework requires research by a subject matter expert for the given input portfolio. In this paper, we propose to replace the conventional spatial interpolation techniques – Kriging, Inverse Distance Weighting (IDW) and Radial Basis Function (RBF) (Burrough et al., 1998) – in the framework of Hejazi et al. (2015) with a neural network. The proposed neural network can learn a good choice of distance function and use the given distance function to efficiently and accurately interpolate the Greeks for the input portfolio of VA contracts. The proposed neural network only requires knowledge of a set of parameters that can fully describe the types of VA contracts in the input portfolio and uses these parameters to find a good choice of distance function.

The rest of this paper is organized as follows. Section 2 provides a brief summary of existing methods for the valuation of portfolios of VA products. The main focus of Section 2 is on the spatial interpolation framework of Hejazi et al. (2015) that has been successful in providing the Greeks for a large portfolio of VA products in an efficient and accurate way. Section 3 describes the neural network framework and provides background information on neural networks. We provide the intuition behind the proposed model and the novel training technique used to calibrate (a.k.a. to train) the network. Section 4 provides insights into the performance of the neural network framework in estimation of Greeks for a large synthetic portfolio of VA contracts. Section 5 concludes the paper with a discussion of our future work and possible applications of the proposed framework.

2. Portfolio valuation techniques

If one thinks of VAs as exotic market instruments (Hull, 2006), the traditional replicating portfolio approach can be used to find the value of a portfolio of VA products. The main idea behind this approach is to approximate the cash flow of liabilities for a portfolio of VA contracts using well-formulated market instruments such as vanilla derivatives. The problem is often formulated as a convex optimization problem where the objective is to minimize the difference between the cash flow of the input portfolio and the replicating portfolio. Depending on the norm associated with the problem, linear programming (Dembo and Rosen, 1999) or quadratic programming (Daul and Vidal, 2009; Oechslin et al., 2007) is used in the literature to find the replicating portfolio. The replicating portfolio, in our application of interest, does not provide us with an efficient alternative to MC simulations, as one still needs to find the cash flow of the input portfolio for each year up to maturity.

Least Squares Monte Carlo (LSMC) regresses the liability of the input portfolio against some basis functions representing key economic factors (Longstaff and Schwartz, 2001; Carriere, 1996). LSMC has been proposed in the literature to reduce the number of inner loop scenarios in nested MC simulations (Cathcart and Morrison, 2009). Depending on the type of embedded guarantees, size of investment and characteristics of the policyholder, VA contracts have a significant number of numeric attributes, each covering a broad range. Therefore, an accurate regression using LSMC requires incorporation of many sample points, and hence is computationally demanding.

Recently, Replicated Stratified Sampling (RSS) (Vadiveloo, 2011) and Kriging based techniques (Gan, 2013; Gan and Lin, 2015) have been proposed to reduce the number of VA contracts that must be included in the MC simulations. Both of these methods, use the Greeks for samples of the input portfolio to estimate the Greeks of the full input portfolio. RSS requires several iterations of sample generation and evaluation to converge to a final result. This makes it more computationally demanding than the Kriging based techniques of Gan (2013) and Gan and Lin (2015) that require MC simulations results for only one sample. We discuss in our earlier paper (Hejazi et al., 2015) how the Kriging based techniques of Gan (2013) and Gan and Lin (2015) can be categorized under a general spatial interpolation framework. The spatial interpolation framework generates a sample of VA contracts from the space in which the VA contracts of the input portfolio are defined. The Greeks for the sample are evaluated using nested MC simulations. The results of MC simulations are then used by a spatial interpolation technique to generate an estimate for the Greeks of the input portfolio.

In Hejazi et al. (2015), we provide numerical and theoretical results comparing the efficiency and accuracy of different conventional spatial interpolation techniques, i.e., Kriging, IDW and RBF. Our results demonstrate that, while the Kriging method provides better accuracy than either the IDW method or the RBF method, it is less efficient and has a lower resolution. By lower resolution, we mean that the Kriging method can provide the Greeks for only the input portfolio in an efficient manner, while both the IDW method and the RBF method approximate the Greeks efficiently for each VA contract in the input portfolio.

3. Neural network framework

As we discuss in our earlier paper (Hejazi et al., 2015), spatial interpolation techniques can provide efficient and accurate estimation of the Greeks for a large portfolio of VA products. Although IDW and RBF methods provide better efficiency and resolution than Kriging methods, they are less accurate than Kriging methods. Our experiments in Hejazi et al. (2015) demonstrate the significance of the choice of distance function on the accuracy of IDW and RBF methods. A manual approach to find the best distance function that minimizes the estimation error of the IDW and the RBF method for a given set of input data is not straightforward and requires investing a significant amount of time. The difficulty