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Optimal capital injection and dividend distribution for growth restricted diffusion models with bankruptcy

Jinxia Zhu[a,*], Hailiang Yang[b]

[a] School of Risk and Actuarial Studies, The University of New South Wales, Kensington Campus, NSW 2052, Australia
[b] Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam Road, Hong Kong

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ABSTRACT

We consider the optimal capital injection and dividend control problem for a class of growth restricted diffusions with the possibility of bankruptcy. The surplus process of a company is modeled by a diffusion process with return and volatility being functions of the surplus process. The company can control the dividend payments and capital injections with the goal of maximizing the expectation of the total discounted dividends minus the total cost of capital injections up to the time of bankruptcy. We distinguish three cases and provide optimality results for each case.

1. Introduction

The optimal dividend control problem has attracted significant interest in the literature; see Albrecher and Thonhauser (2009), Avanzi (2009), Schmidli (2008) and the references therein. Many works model the underlying surplus process by a Brownian motion with drift (see for example, Asmussen et al., 2000; Guo et al., 2004; Yang et al., 2005; Cadenillas et al., 2006; He and Liang, 2009). The dividend optimization problem for more general diffusions are studied in Shreve et al. (1984), Højgaard and Taksar (2001), Bäuerle (2004) and Alvarez and Virtanen (2006), Cadenillas et al. (2007), Paulsen (2008), Zhu (2015) and references therein.

The dividend optimization problem with the inclusion of capital injections which aims at maximizing the expected total discounted dividend payments minus the expected total discounted costs of capital injections is studied in Shreve et al. (1984) and has gained much interest in the recent literature. Shreve et al. (1984) investigated this optimization problem (framed as a reflection problem in the paper) for a general diffusion model subject to the constraint that the surplus process remains non-negative all the times (guaranteed via capital injections whenever necessary even though this may not be optimal in some situations). Løkka and Zervos (2008), however, addressed the optimal dividend and issuance of equity policies control problem with the possibility of bankruptcy for a Brownian motion model. He and Liang (2008) studied a similar problem with the addition of proportional reinsurance policy for the Brownian motion model. Meng and Siu (2011) applied the viscosity solution approach to studying the optimal capital injection and dividend control problem for the Brownian motion model where there are fixed and proportional costs for each dividend payment. Sethi and Taksar (2002) addressed the optimal dividend and financing control problem for a more general diffusion model. However, the paper does not taking into consideration of the possibility of bankruptcy (which generally occurs when the surplus drops below a certain level, say 0) at all.

This paper studies the optimal capital injection and dividend control for a class of growth restricted diffusion models with the possibility of bankruptcy. As in Løkka and Zervos (2008), we assume that the objective is to maximize the expected discounted dividend payments minus the expected discounted costs of capital injections up to the time of ruin, which is defined to be the moment that the surplus process drops below 0 for the first time. Our work can be considered as a generalization of the control problem

* Corresponding author.
E-mail addresses: jinxia.zhu@unsw.edu.au (J. Zhu), hlyang@hku.hk (H. Yang).

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in Løkka and Zervos (2008) in that both the drift and diffusion coefficients of the diffusion model in our paper are functions of the level of the surplus and therefore the model in our paper includes the Brownian motion model considered in Løkka and Zervos (2008) as a special case. The major technical difficulty in our extended model is caused by the fact that the ordinary differential equation (ODE) involved in the associated Hamilton–Jacobi–Bellmen (HJB) equation, unlike the constant coefficient ODE in Løkka and Zervos (2008), has varying coefficients that are general functions (with unspecified forms) of the variable. This means that we will not be able to derive the explicit form of the solution, let alone to obtain a simple exponential form that the solution in Løkka and Zervos (2008) has. The explicit and especially exponential form in Løkka and Zervos (2008) allows the authors to derive analytical properties directly, which plays a crucial role in finding the final optimal results.

We organize the rest of the paper as follows. In Section 2, we provide the formulation of the optimization problem. In Section 3, we study the functions that are solutions to the ordinary differential equation involved in the associated HJB equation and some functions constructed from these solutions. We distinguish and analyze 3 cases, and present the optimality results for each case in Section 4. We illustrate the results with two examples in Section 5. Concluding remarks are provided in Section 6.

2. Problem formulation

Consider a probability space $(\Omega, \mathcal{F}, \mathrm{P})$. Let $\{W_t; t \geq 0\}$ be a standard Brownian motion and $\{\mathcal{F}_t; t \geq 0\}$ be the minimal complete σ-field generated by the stochastic process $\{W_t; t \geq 0\}$. Let X_t denote the cash flow surplus at time t of a company in absence of capital injections and dividend payments. Assume that the initial value of the surplus process, X_{0-}, is $\mathcal{F}_0$ measurable, and that X_t has the following dynamics

$$\mathrm{d}X_t = \mu(X_{t-})\mathrm{d}t + \sigma(X_{t-})\mathrm{d}W_t, \quad t \geq 0, \tag{2.1}$$

where the functions $\mu(\cdot)$ and $\sigma(\cdot)$ are Lipschitz continuous, differentiable and grow at most linearly on $[0, \infty)$. Let δ denote the force of interest for the valuation of shareholders' cash flows. Furthermore, we assume that the function $\sigma(\cdot)$ is positive and non-vanishing, and $\mu'(x) < \delta$ for $x \geq 0$.

Remark 2.1. The diffusion under the constraint, $\mu'(x) < \delta$ for $x \geq 0$, is general compared with most of the models used in the literature of the dividend optimization problem with or without the inclusion of capital injection control. In the literature, most of the works used the drifted Brownian motion model (i.e., $\mu(\cdot) =$ a constant and $\sigma(\cdot) =$ a constant), a couple of papers considered the Brownian model compounded by a constant force of interest (i.e. $\mu(x) = p + rx$ with $p \geq 0$ and $r < \delta$, $\sigma(x) =$ a constant) and one paper investigated the mean-reverting process (i.e. $\mu(\cdot) = c - rx$; see Cadenillas et al., 2007). All these are special cases of the growth restricted diffusions considered in this paper.

The company can distribute part of its assets to the shareholders as dividends and the shareholders can reinvest (under no obligation) via capital injections. There are transaction costs associated with dividend payments and capital injections. For each dollar of reinvestment, it includes $c(c > 0)$ dollars of transaction cost and hence leads to an increase of $1 - c$ dollars in the surplus through capital injections. Let C_t denote the cumulative amount of capital injections up to time t. Then the total cost for capital injections up to time t is $\frac{C_t}{1-c}$. For each dollar of dividends received by the shareholders, there will be $d(d > 0)$ dollars of transaction cost. Let D_t denote the cumulative amount of dividends paid out by the company up to time t. Then the total amount of dividends received by the shareholders up to time t is $\frac{D_t}{1+d}$. Both $\{C_t; t \geq 0\}$ and $\{D_t; t \geq 0\}$ are controllable by the company. We call $\pi := \{(C_t, D_t); t \geq 0\}$ a control strategy.

The dynamics of the controlled surplus process (by the strategy π) is

$$\mathrm{d}X_t^\pi = \mu(X_{t-}^\pi)\mathrm{d}t + \sigma(X_{t-}^\pi)\mathrm{d}W_t - \mathrm{d}D_t + \mathrm{d}C_t, \quad t \geq 0. \tag{2.2}$$

Definition 2.1. A strategy $\pi = \{(C_t, D_t); t \geq 0\}$ is said to be *admissible* if (i) both $\{C_t; t \geq 0\}$ and $\{D_t; t \geq 0\}$ are nonnegative, increasing, càdlàg, and $\{\mathcal{F}_t; t \geq 0\}$-adapted processes, (ii) $C_{0-} = D_{0-} = 0$, and (iii) $\Delta D_t \leq X_t^\pi$.

We use Π to denote the class of admissible strategies.

Define the time to bankruptcy by

$$T^\pi = \inf\{t \geq 0 : X_t^\pi < 0\}.$$

Note that bankruptcy may never occur under some strategies. For example, if a company injects enough capital whenever the surplus process is about to drop below 0 to keep the surplus process at or above 0, bankruptcy never occurs. We define $T^\pi = +\infty$ in this case.

Define

$$\mathrm{P}_x(\,\cdot\,) = \mathrm{P}(\,\cdot\,|X_{0-} = x), \qquad \mathrm{E}_x[\,\cdot\,] = \mathrm{E}[\,\cdot\,|X_{0-} = x].$$

The performance of a control strategy π is measured by the return function defined as follows:

$$R_\pi(x) = \mathrm{E}_x\left[\int_{0-}^{T^\pi} \frac{e^{-\delta t}}{1+d}\mathrm{d}D_t - \int_{0-}^{T^\pi} \frac{e^{-\delta t}}{1-c}\mathrm{d}C_t\right], \quad x \geq 0. \tag{2.3}$$

Remark 2.2. (i) From the above definition, we can see that the class of admissible strategies, Π, includes admissible strategies under which no capitals will be injected at all and strategies that inject capitals before the surplus falls below 0 so that bankruptcy will never occur. For example, the strategy that prescribes to inject no capitals at all and to pay out the excess of surplus over a pre-specified non-negative number as dividends is an admissible strategy. Under such strategy, the controlled surplus will fall below 0 eventually and therefore the bankruptcy time is finite. Another special admissible strategy is to distribute all the available surplus as dividends at time 0 and inject no capitals at all. In this case, bankruptcy occurs immediately at time 0, and the associated return function is $\frac{x}{1+d}$.

(ii) We can see that in our paper the company is not compelled to inject capitals at any time, unlike in Kulenko and Schmidli (2008) where the controlled surplus is never allowed to be negative, which is guaranteed via compulsory capital injections.

For convenience, we use X and X^π to denote the stochastic processes $\{X_t; t \geq 0\}$ and $\{X_t^\pi; t \geq 0\}$, respectively. We can see that for any admissible strategy π, the stochastic process X^π is right-continuous and adapted to the filtration $\{\mathcal{F}_t; t \geq 0\}$.

The objective of this paper is to study the maximal return function (also called value function):

$$V(x) = \sup_{\pi \in \Pi} R_\pi(x), \; x \geq 0, \tag{2.4}$$

investigate the existence of optimal strategies and identify an optimal admissible strategy, if any.