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Pricing and hedging of guaranteed minimum benefits under regime-switching and stochastic mortality[☆]

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ABSTRACT

This paper presents a novel framework for pricing and hedging of the Guaranteed Minimum Benefits (GMBs) embedded in variable annuity (VA) contracts whose underlying mutual fund dynamics evolve under the influence of the regime-switching model. Semi-closed form solutions for prices and Greeks (i.e. sensitivities of prices with respect to model parameters) of various GMBs under stochastic mortality are derived. Pricing and hedging is performed using an accurate, fast and efficient Fourier Space Time-stepping (FST) algorithm. The mortality component of the model is calibrated to the Australian male population. Sensitivity analysis is performed with respect to various parameters including guarantee levels, time to maturity, interest rates and volatilities. The hedge effectiveness is assessed by comparing profit-and-loss distributions for an unhedged, statically and semi-statically hedged portfolios. The results provide a comprehensive analysis on pricing and hedging the longevity risk, interest rate risk and equity risk for the GMBs embedded in VAs, and highlight the benefits to insurance providers who offer those products.

1. Introduction

The variable annuity (VA) markets have grown dramatically over the past decades as a response to a growing demand for products that can manage longevity risk by an ageing population. A VA is a long-dated contract between the policyholder and an insurance company under which the policyholder makes either a single premium or a stream of periodic premium payments during the accumulation phase. In return, the insurer guarantees minimum periodic payments starting either immediately or at a future date. VAs provide certainty in income provisions as well as allowing the policyholder to gain exposure to equity markets by linking the level of payments to the performance of a chosen investment fund. Insurers often offer guarantees embedded in VAs, such as the Guaranteed Minimum Benefits (GMBs), to restrict the downside risk, thus, making these products more appealing to potential annuitants. Furthermore, some countries (e.g. US and Canada) provide tax-shelters for the investment gains of VAs. These attractive features drive the increasing interest in VAs amongst an ageing population.

The GMBs embedded in VAs can be classified into two main categories, namely, the guaranteed minimum death benefits (GMDBs) and the guaranteed minimum living benefits (GMLBs). In a GMDB contract, the policyholder's beneficiaries are paid the contracted amount in the event of the policyholder's untimely death whilst the contract is still in force. The benefits paid to beneficiaries are usually based on the premium payments and the performance of the underlying mutual fund. GMLBs offer living protection against market risk through guaranteeing a variety of benefits which can be classified as the GMxB where "x" stands for maturity (M), income (I) and withdrawal (W). This paper primarily focuses on pricing and hedging of GMMBs, GMIBs, and GMDBs as they all have pre-specified maturity dates,[1] and can be classified as path-independent options.

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[1] This is different to the GMWB, which lasts for life, thus, the maturity date is uncertain.

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A GMMB is a guarantee that provides the policyholder with a minimum benefit on maturity of the contract, while a GMIB is a guarantee that provides the policyholder with a minimum amount of income stream for a given period of time when a policyholder annuitizes, regardless of the performance of the underlying investment. A GMWB is a guarantee that allows the policyholder to recoup at least the initial investment amount by periodically withdrawing a portion of the entire investment. Any excess in the investment account is paid at maturity.

Insurance providers are faced with several types of risks when offering VAs. First of all, due to a long-term nature of these products and mortality improvements that have been faster than anticipated in the past, it is essential for the insurance providers to effectively capture the mortality risk. Secondly, given that the guarantees can be written in nominal amounts, it is essential to incorporate the interest rate risk in order to correctly account for the evolution of interest rates during the lifetime of the contract. Furthermore, the insurance providers are also exposed to the equity risk when guaranteeing a minimum level of return to their policyholders. To account for all three types of risks, one requires to develop a realistic modelling framework. Unfortunately, reinsurance is not a viable risk management strategy for GMBs: the risk premiums are too high and continue to increase as reinsurers became more aware of the risks embedded in these guarantees. Furthermore, following the Global Financial Crisis, the reinsurers no longer offer coverage for GMWBs (Hyndman and Wenger, 2013). Due to the increasing awareness of the risks associated with GMBs in VAs and the decreasing availability of reinsurance, insurers and superannuation providers are required to develop better internal risk management systems.

The GMBs pricing framework often consists of using a financial model to capture the equity and interest rate risk, and a mortality model to capture the mortality risk. This is done with the reasonable assumption that the mortality process is independent of the financial markets (see e.g. Ulm, 2013; Fung et al., 2014; Da Fonseca and Ziveyi, 2015). Most of the previous literature on VAs has modelled the underlying fund dynamics using the standard geometric Brownian motion (GBM) process, see e.g. Brennan and Schwartz (1976), Milevsky and Salisbury (2006), Bauer et al. (2008), Piscopo and Haberman (2011) and Fung et al. (2014). Some studies have either ignored the mortality effects (Milevsky and Salisbury, 2006; Dai et al., 2008; Chen et al., 2008), or used deterministic mortality rates (Milevsky and Promislow, 2001; Milevsky and Posner, 2001; Bauer et al., 2008). However, given the long-term nature of variable annuity contracts it is imperative to value these products using a realistic framework for the underlying fund dynamics (Coleman et al., 2007) and also effectively capture mortality improvements and volatility at older ages (Biffis, 2005; Cairns et al., 2006; Blackburn and Sherris, 2013). There is a significant amount of research focusing on the development of continuous-time mortality models (see Dahl, 2004; Biffis and Milossovich, 2006 and Biffis, 2005). Applications of stochastic mortality processes have been further discussed in Luciano and Vigna (2005), Biffis (2005), Schrager (2006), Piscopo and Haberman (2011), Blackburn and Sherris (2013) and Fung et al. (2014) and Da Fonseca and Ziveyi (2015) among others. In addition to stochastic mortality, several authors introduce stochastic interest rates (Bacinello et al., 2011; Krayzler et al., 2012) and stochastic volatility (Bacinello et al., 2011; Da Fonseca and Ziveyi, 2015) into the modelling framework.

This paper presents a hybrid framework for pricing and hedging the GMBs, which include GMMBs, GMIBs and GMDBs, under the regime-switching framework for the fund dynamics and stochastic mortality. A multi-factor stochastic mortality process (Blackburn and Sherris, 2013) is incorporated to account for the uncertain future mortality experiences on the VA portfolios.

When implementing the valuation framework, we adopt the Fourier Space Time-stepping (FST) method (Jackson et al., 2008) that utilises the fast Fourier transform (FFT) algorithm (see Carr and Madan, 1999 who first introduce the FFT to option pricing), a proven computational tool which is fast and efficient in generating guarantee values and the associated hedge ratios. The usage of FFT algorithms is also prevalent in regime-switching models as presented in Liu et al. (2006), who show that the solutions obtained via the FFT approximate the true value with sufficient accuracy. Shen et al. (2014) utilise the FFT to price European options under a double-regime switching model. Da Fonseca and Ziveyi (2015) value GMMB and GMDB guarantees embedded in VAs whose underlying fund consists of several assets using the FFT. Jackson et al. (2008) emphasise the versatility of the FST method in pricing path-independent and path-dependent options. Surkov and Davison (2010) extend the use of the FST algorithm by demonstrating its applicability in computing the Greeks of options which can be used for hedging purposes. Lippa (2013) prices GMWBs under the GBM using the FST algorithm, and demonstrates that its numerical results are consistent with those found in Chen et al. (2008). To our knowledge, Lippa (2013) is the first and only research paper which has utilised the FST algorithm in pricing GMBs. This paper marks the second yet, portraying a more sophisticated pricing framework of GMBs under the regime-switching environment as well as stochastic mortality. This paper also pioneers the use of the FST algorithm in computing the Greeks, or sensitivities of GMB that are utilised when implementing static and semi-static hedging strategies. It provides a toolkit which can be used by practitioners to effectively mitigate the risks when offering GMBs and their synthetic products.

The contribution of this paper can be summarised as follows; (i) we develop a valuation framework for GMBs under the regime-switching Log-Normal model; (ii) we perform sensitivity analysis with respect to model parameters; (iii) we analyse static and semi-static hedging strategies and their effectiveness. In so doing, the paper extends the existing literature on generalised pricing frameworks for VAs (e.g. Bauer et al., 2008 who assumes the GBM model for the fund and deterministic mortality rates). Furthermore, Bauer et al. (2008) solve the resulting valuation expressions with the aid of Monte Carlo simulation, which is significantly slower computationally as compared to the FST algorithm. Finally, considering that the Australian annuities market is rather underdeveloped, we utilise Australian mortality data to shed more insights on the risk profile of GMBs, thus, contributing by quantifying the risks embedded in GMBs for Australian insurers and super providers.

The remainder of the paper is organised as follows: Section 2 introduces the modelling framework, which includes the financial and mortality models, as well as an FST algorithm utilised for valuation of the products. Section 3 provides a general pricing and hedging framework for GMMBs, GMIBs and GMDBs. Numerical results, which include sensitivity analysis with respect to model parameters, as well as hedging performance are summarised in Section 4. Section 5 concludes and provides final remarks.

2. Modelling framework

2.1. Regime-switching models

A regime switching framework is adopted in modelling the financial market. Since the market is incomplete under the real world probability measure, $\mathbb{P}$, the process of completing the market under the regime switching environment is accomplished by applying Esscher transforms (Gerber and Shiu, 1994)[2] which

[2] We acknowledge that outside of the actuarial literature, there are alternative methods to the Esscher transform. A popular alternative is the minimal martingale measure (Föllmer and Schweizer, 1991) whose aim is to minimise the distortion in the probability space, when moving from $\mathbb{P}$ to $\mathbb{Q}$.