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A micro-level claim count model with overdispersion and reporting delays

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ABSTRACT

The accurate estimation of outstanding liabilities of an insurance company is an essential task. This is to meet regulatory requirements, but also to achieve efficient internal capital management. Over the recent years, there has been increasing interest in the utilisation of insurance data at a more granular level, and to model claims using stochastic processes. So far, this so-called 'micro-level reserving' approach has mainly focused on the Poisson process.

In this paper, we propose and apply a Cox process approach to model the arrival process and reporting pattern of insurance claims. This allows for over-dispersion and serial dependency in claim counts, which are typical features in real data. We explicitly consider risk exposure and reporting delays, and show how to use our model to predict the numbers of Incurred-But-Not-Reported (IBNR) claims. The model is calibrated and illustrated using real data from the AUSI data set.

1. Introduction

The financial liability of insurers due to outstanding claims typically represents more than half of the company's total liabilities, and a factor of its economic capital. Its accurate estimation is thus of paramount importance. Some of the complexity of this *reserving* problem is due to reporting delays, leading to claims that have occurred, but have not been reported yet ("IBNR"). In this paper, we are interested in estimating the number of IBNR claims using micro-level data.

Nowadays, insurers record detailed information for each individual claim, which may include, for example, arrival and reporting dates of a claim, as well as the date and amount of each transaction. This is what we call a micro-level data set. If, on the other hand, information is aggregated over a (small) number of (long) discrete time periods, then data is qualified as 'macro-level'. This is the case, for instance, of loss reserving triangles. A vast majority of the literature on modelling insurance claims is based on macro-level claims data, including the Mack's chain-ladder model (Mack, 1993), where a typical choice is to use a random variable to model each data point of aggregated observations. For more examples, one can refer to Taylor (2000) and Wüthrich and Merz (2008).

A micro-level approach can arguably present advantages over a macro-level approach. Firstly, the aggregation of information may lead to the disappearance of useful, perhaps material information. For example, information of the arrival and reporting time of each individual claim (and their trends) may be critical for the quality of a model. Secondly, parameter uncertainty of a macro-level model can be high due to a small number of observations (England and Verrall, 2002), resulting in less predictive power. Some of the early theoretical work in modelling micro-level claims arrival and development can be traced back to Arjas (1989) and Norberg (1993, 1999, who adopted a marked Poisson process approach). In recent years, Antonio and Plat (2014) and Larsen (2007) have further implemented Norberg's framework with real data sets. Moreover, Pigeon et al. (2013) and Pigeon et al. (2014) develop a discrete time framework, whereby numbers of claims follow Poisson distributions. Besides the papers that study the

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overall micro-level claims modelling, Jewell (1989), Zhao et al. (2009), Zhao and Zhou (2010) have focused on the issue of modelling the claims arrival with a Poisson process as well as the reporting delay distribution, while Taylor et al. (2008) model individual claims development using case estimates as additional information.

The natural choice of methodology to model micro-level data is to use a continuous stochastic process. The classical model for claims processes is the Poisson process (see, e.g., Mikosch, 2006), under which the average number of claims per time unit is a constant λ.

An alternative and more general approach is to adopt a deterministic function $\lambda(t)$ instead to model the claim intensity, which results in an inhomogeneous Poisson process. The increased flexibility permits an approach that more accurately represents the nature of claim frequencies in practice where the intensity is not stationary. However, it still does not capture overdispersion, which is frequently observed in claim counts data (for example, see Section 2.9 of de Jong and Heller, 2008) and such a deterministic intensity does not allow for serial dependency of claims counts (see Denuit et al., 2007).

The issues mentioned above can be solved by modelling the intensity as a non-negative stochastic process. This results in a *doubly stochastic Poisson process*, or *Cox process* (see, for example Cox, 1955; Lando, 1998). Doubly stochastic Poisson processes have been widely applied in varying research areas, such as finance (Frey and Runggaldier, 2001), credit risk modelling (Lando, 1998), risk theory (Björk and Grandell, 1988; Albrecher and Asmussen, 2006), mortality modelling (Biffis, 2005; Schrager, 2006), catastrophe modelling (Dassios and Jang, 2003; Jang and Fu, 2012), insurance claim modelling (Avanzi et al., 2016; Badescu et al., 2015, 2016), reinsurance pricing (Dassios and Jang, 2005) and operational risk modelling (Jang and Fu, 2008).

There are a number of possible choices for the intensity process under the Cox process approach, for example, a diffusion process (Frey and Runggaldier, 2001; Schrager, 2006), a continuous time Markov chain (Frey and Runggaldier, 2001), a discrete time process with state-dependent (Erlang) intensities (Badescu et al., 2015, 2016), a jump-diffusion process (Biffis, 2005) or a shot noise process (Dassios and Jang, 2003, 2005; Albrecher and Asmussen, 2006). In this paper, we focus our illustration using the shot noise process, which processes a number of attractive behaviours such as tractability and mean reverting intensity.

A complication arising from the use of a Cox process lies in its estimation. When a (homogeneous) Poisson process is assumed, standard likelihood techniques are available (see, for example, Mikosch, 2006). However, the maximum likelihood estimation approach is in general not directly applicable to a Cox process. This is because the arrivals of insurance claims are not independent under the shot noise assumption. Secondly, although the complete likelihood of observing both the Cox process and the shot noise process is simple to derive, the likelihood of observing the Cox process unconditionally on the shot noise process involves a high dimensional integral, which is not computationally efficient to calculate. Furthermore, the prediction of the IBNR counts under a Cox process model also requires the estimation of the unobservable intensity. For all those reasons, the development of a filtering algorithm is necessary.

In the case of a shot noise intensity, two filtering methods have been proposed in the literature. One method is to use a Kalman filter, which involves Gaussian approximation and is suitable in the case with high frequency but low impact shots (Dassios and Jang, 2005). The other method is to use a Reversible Jump Markov Chain Monte Carlo ("RJMCMC") filter (Centanni and Minozzo, 2006a,b), which is based on RJMCMC simulations of the shot noise trajectory. A comparison of these two methods can be found in Avanzi et al. (2016). However, the actual implementation of the model to insurance data is not straightforward, and is generally not discussed in the existing actuarial literature. In particular, the frequency of claims is subject to exposure and reporting delays. These require non trivial model extensions. The filtering algorithm must also be modified in order to allow for such features. In this paper, we address these issues in the model construction and estimation.

This paper is structured as follows. The model assumptions along with some of the main theoretical properties are introduced in Section 2. We consider estimation methods in detail in Section 3, and extend the existing methodology to allow for varying risk exposure, and for reporting delays. We illustrate the procedures and performance of the estimation and prediction algorithms with a simulated dataset in Section 4. Furthermore, we calibrate our model to the AUSI (real) insurance data set in Section 5, and provide prediction results.

For convenience, a table with notation used throughout the paper is provided in Appendix D.

2. A shot noise Cox process with exposure and reporting delays

In this section, we develop the shot noise Cox model that is considered in this paper. Section 2.1 reviews the stationary shot noise Cox model without reporting delays. Section 2.2 defines an appropriate non-stationary version in order to allow for exposure changes over time, and Section 2.3 explains how reporting delays can be incorporated.

2.1. A stationary shot noise Cox process model

Model assumption 2.1.1 (*Cox Process, Grandell, 1976*)**.** Denote by $N(t)$ the number of claims up to time t. We assume that $\{N(t), t \geq 0\}$ is a Cox process, that is, there exist a non-negative stochastic process $\{\Lambda(t), t \geq 0\}$ and a homogeneous Poisson process with intensity rate 1, $\{\tilde{N}(t), t \geq 0\}$, such that $\{N(t), t \geq 0\}$ has the same distribution as $\tilde{N} \circ \left(\int_0^t \Lambda(s)ds\right)$.

A Cox process (Model assumption 2.1.1) can be interpreted as an extension of a Poisson process, where the intensity process $\{\Lambda(t), t \geq 0\}$ is stochastic. In particular, we assume that the stochastic intensity, $\Lambda(t)$, is a shot noise process (see, for example, Jang, 2004; Centanni and Minozzo, 2006a,b).

Model assumption 2.1.2 (*Homogeneous Shot Noise Intensity Process*)**.** The stochastic intensity $\Lambda(t)$ is a stationary shot noise process if

$$\Lambda(t) = \Lambda(0)e^{-kt} + \sum_{j=1}^{J(t)} X_j e^{-k(t-\tau_j)}, \quad t \geq 0, \tag{2.1}$$

where τ_j represents the arrival time of the shots resulting from a homogeneous Poisson process $J(t)$ with a deterministic intensity ρ, and the shots X_j's are independent and identically distributed random variables with a density function f_X (on the positive domain with finite mean). Furthermore, we assume that $\Lambda(0)$, the initial level of $\Lambda(t)$, follows the stationary distribution of the shot noise process. This is a convenient assumption and ensures that the shot noise process is stationary from $t = 0$.

Remark 2.1. In this paper, we follow the literature (see, for example, Dassios and Jang, 2003; Jang, 2004; Dassios and Jang, 2005; Centanni and Minozzo, 2006a,b; Avanzi et al., 2016) and consider an exponential decay function. However, it should be noted that alternative decay functions can be considered, at a slight loss of mathematical convenience (see, Schmidt, 2014, for details).