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Catastrophe equity put options with target variance

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ABSTRACT

In this study, we consider a new class of catastrophe equity put options, whose payoff depends on the ratio of the realized variance of the stock over the life of the option and the target variance, which represents the insurance company's expectation of the future realized variance. This kind of options could help insurance companies raise more equity capital when a large number of catastrophic events occur during the life of the option. We employ a compound doubly stochastic Poisson process with lognormal intensity to describe accumulated catastrophe losses and assume the volatility varies stochastically. Finally, numerical results are presented to investigate the values of this class of options.

1. Introduction

With an increasing number of catastrophic events, insurance companies suffer from substantial financial losses. To increase the capacity for catastrophe coverage and manage catastrophe risk, related innovative financial instruments including catastrophe bonds and catastrophe equity put options have been introduced in the capital markets. These financial instruments are generally called as catastrophe derivatives and now play an important role in catastrophe risk management. Catastrophe bonds allow the issuer to forgive full or partial principal in order to cover catastrophe losses, once a catastrophic event occurs. The holders of catastrophe equity put options have the right to sell a specified amount of the stocks at a predetermined price. By trading catastrophe derivatives, insurance companies could raise additional funds to pay for catastrophe losses.

To value catastrophe derivatives, the accumulated losses are needed to be specified. In the literature, a Poisson process is usually used to capture the number of catastrophe loss occurrences. For instance, Jaimungal and Wang (2006) price catastrophe equity put options with the losses described by a compound Poisson process. Lin et al. (2009) adopt a doubly stochastic Poisson process with lognormal intensity to describe the number of catastrophe loss occurrences. In addition, the lognormal intensity is proposed based on the annual number of catastrophic events adjusted by Commerce Census Fixed Weighted Construction Cost index and population in the United States from 1950 to 2004. Wang (2016) investigates the valuation of catastrophe equity put options with default risk by assuming the number of catastrophic events follows a doubly stochastic Poisson process. Additionally, the author considers default risk using typical structural approaches as in Klein (1996) and Wang (forthcoming). Lee and Yu (2007) focus on how a reinsurance company can reduce its default risk by issuing catastrophe bonds. On the other hand, Chang and Hung (2009) employ a Lévy process with finite activity to describe the underlying asset price and obtain explicit analytical formulae for catastrophe equity put option prices under deterministic and stochastic interest rates, respectively. Yu (2015) investigates the values of catastrophe equity put options by assuming that stock processes follow an exponential jump–diffusion process with jump terms described by two compound Poisson processes, corresponding to the catastrophe loss process and other financial market risks, respectively.

Another strand of the literature studies the distribution of the loss size. Among them, Levi and Partrat (1991) show that the Poisson lognormal model performs best using the data on hurricanes observed in the United States between 1954 and 1986. Burnecki and Kukla (2003) apply chi-squared test, Kolmogorov–Smirnov (KS), Cramer–von Mises (CM) and Anderson–Darling (AD) tests to compare heavy-tailed distributions including lognormal distribution, Pareto distribution, Burr distribution, and Gamma distribution. The results corresponding to the data on the United States

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market's loss amounts between 1990 and 1999 demonstrate the lognormal distribution passes all tests. Braun (2011) fits various parametric distributions to normalized historical loss data for hurricanes and earthquakes in the United States between 1900 and 2005, showing the Burr distribution is the most adequate representation for loss severities. The empirical literature motivates researchers to present numerical experiments to illustrate the prices of catastrophe derivatives using heavy-tailed distributions. For example, a lognormal distribution is used by Lin et al. (2009), Lo et al. (2013) and Wang (2016), while Jaimungal and Wang (2006) adopt a Gamma distribution.

In this paper, we study a new class of catastrophe equity put options. Insurance companies have to cover a large number of catastrophe losses, when lots of catastrophic events occur in the future. Otherwise, fewer catastrophe losses are needed to cover. The amount of equity capital insurance companies need to raise increases with catastrophe losses, and a large number of catastrophe losses correspond to a high value of the variance of the stock. Motivated by these facts, we consider a new kind of catastrophe equity put options, which allow insurance companies to take a joint exposure to the evolution of the stock and to the realized variance or volatility as well. Basically, we investigate catastrophe equity put options with target variance, whose payoff depends on the ratio of the realized variance of the stock over the life of the option and the target variance, which represents the insurance company's expectation of the future realized variance. Holding this kind of options, insurance companies could raise more equity capital when more catastrophic events occur during the life of the option. This kind of options are similar to target volatility options (see, e.g., Di Graziano and Torricelli (2012), Wang and Wang (2014), and Grasselli and Marabel Romo (2015)). To manage catastrophe risk, insurance and reinsurance companies have been using innovative financial instruments such as catastrophe bonds and catastrophe equity put options in the capital markets since the early 1990s. Recently, the market for catastrophe equity put options has become extinct. Therefore, it is quite important to introduce some new kinds of products in the capital markets. This paper attempts to design a new class of products and mainly focuses on catastrophe equity put options with target variance; however, the product considered in this paper may be less applicable,[1] which motivates us to consider more applicable products in the future.

The remainder of this paper is organized as follows. In Section 2, the framework is proposed and the new kind of catastrophe equity put options are considered. Section 3 presents numerical results. Finally, concluding remarks are contained in Section 4.

2. The model

In this section, we describe the theoretical framework for valuing catastrophe equity put options with target variance, which allow investors to take a joint exposure to the evolution of the asset and to its realized variance as well. Compared with vanilla catastrophe equity put options, this kind of options can help insurance companies to raise more equity capital when the number of catastrophe loss occurrences is larger.

In order to price such options, we need to specify the accumulated losses and the dynamics of the underlying asset. Following Biagini et al. (2008) and Wu and Chung (2010), all processes including interest rate, catastrophe losses and the stock price are assumed directly under the risk-neutral measure Q on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, Q)$. The assumptions are in order.

[1] The author thanks the referee for pointing out this issue.

2.1. Stochastic interest rate

Since the life time of catastrophe equity put options could be 5 years or more, stochastic interest rate case has been considered in the literature when pricing catastrophe derivatives. For instance, Jaimungal and Wang (2006) extend the framework in Cox et al. (2004) by incorporating stochastic interest rate, which is assumed to satisfy Vasicek's model in Vasicek (1977). However, the value of interest rate may be negative in Vasicek's model. On the other hand, the square-root process in Cox et al. (1985) is also commonly adopted to capture stochastic nature of interest rate and this process can avoid the possible negative interest rate. Here similarly to Wu and Chung (2010), we assume that interest rate is controlled by the following process under the risk-neutral pricing measure Q,

$$\mathrm{d}r(t) = \theta_r(m_r - r(t))\mathrm{d}t + \sigma_r\sqrt{r(t)}\mathrm{d}Z(t),$$

where θ_r denotes the constant reversion rate, m_r represents the long-run mean of the interest rate, σ_r is the instantaneous volatility of the interest rate, and $Z(t)$ is a standard Brownian motion, representing unanticipated instantaneous changes of the interest rate.

2.2. Catastrophe losses

In this subsection, the accumulated losses are specified. Denote by $N(t)$ the total number of catastrophe loss occurrences up to time t and suppose that $N(t)$ follows a doubly stochastic Poisson process with intensity $\lambda(t)$. Similarly to Lin et al. (2009), we adopt the following lognormal intensity $\lambda(t)$ to capture the arrival rate of catastrophic events,

$$\lambda(t) = \lambda(0)e^{\mu_\lambda t - \frac{1}{2}\sigma_\lambda^2 t + \sigma_\lambda W_\lambda(t)}, \tag{2.1}$$

where μ_λ denotes the instantaneous change rate of the arrival rate of catastrophic events, σ_λ is the volatility of change rate and $W_\lambda(t)$ represents a standard Brownian motion on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, Q)$, independent of $Z(t)$. The exponential form of intensity $\lambda(t)$ is motivated by the data on adjusted numbers of natural catastrophes in the United States from 1950 to 2004, which shows an increasing exponential trend in the frequency of catastrophic events (see, e.g., Lin et al., 2009).

Next, we focus on the size of the loss when a catastrophic event happens. It is assumed that the size of the loss is controlled by a random variable. Let $\{l_i, i = 1, 2, \ldots\}$ be the size of the ith loss and we also suppose that $\{l_i, i = 1, 2, \ldots\}$ are independently and identically distributed. Based on the above assumptions, we have the accumulated losses up to time t, denoted by $L(t)$, as follows

$$L(t) = \sum_{i=1}^{N(t)} l_i. \tag{2.2}$$

In a word, catastrophe losses $L(t)$ are described by a compound doubly stochastic Poisson process with lognormal intensity, which reduces to a compound pure Poisson process, if the intensity $\lambda(t)$ is constant, that is, $\mu_\lambda = \sigma_\lambda = 0$.

2.3. Asset value

Now we describe the dynamics of the underlying stock. Once catastrophic events occur, insurance companies have to pay for catastrophe losses, and hence the value of the stock reduces. Here we suppose that the price of the underlying stock is affected by the catastrophe losses $L(t)$ in (2.2) and under the risk-neutral measure