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Uniform asymptotics for a multi-dimensional time-dependent risk model with multivariate regularly varying claims and stochastic return

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ABSTRACT

This paper is devoted to asymptotic analysis for a multi-dimensional risk model with a general dependence structure and stochastic return driven by a geometric Lévy process. We take into account both the dependence among the claim sizes from different lines of businesses and that between the claim sizes and their common claim-number process. Under certain mild technical conditions, we obtain for two types of ruin probabilities precise asymptotic expansions which hold uniformly for the whole time horizon.
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1. Introduction

Consider an insurer which simultaneously operates d ($d \geq 1$) lines of businesses with a common claim-number process. In practice, the insurer will usually invest its wealth in some risky financial products. We describe the price process of its investment portfolio as a geometric Lévy process $\left\{e^{R(t)}; t \geq 0\right\}$ with $\{R(t); t \geq 0\}$ being a Lévy process which starts from 0 and has independent and stationary increments. This assumption on price processes has been extensively used in mathematical finance and insurance; see Wang and Wu (2001), Yuen et al. (2006), Paulsen (2008), and Tang et al. (2010), among many others.

The insurer's surplus process can be expressed by the following d-dimensional risk model:

$$\begin{pmatrix} U_1(t) \\ \vdots \\ U_d(t) \end{pmatrix} = \begin{pmatrix} \rho_1 x e^{R(t)} \\ \vdots \\ \rho_d x e^{R(t)} \end{pmatrix} + \begin{pmatrix} c_1 \int_0^t e^{R(t-s)} ds \\ \vdots \\ c_d \int_0^t e^{R(t-s)} ds \end{pmatrix} - \begin{pmatrix} \sum_{i=1}^{N(t)} X_{1i} e^{R(t-\tau_i)} \\ \vdots \\ \sum_{i=1}^{N(t)} X_{di} e^{R(t-\tau_i)} \end{pmatrix}, \quad t \geq 0, \tag{1.1}$$

where $\{(U_1(t), \ldots, U_d(t)); t \geq 0\}$ denotes the multi-dimensional surplus process, $(\rho_1 x, \ldots, \rho_d x)$ the vector of initial surpluses assigned to different businesses with positive $\rho_1, \ldots, \rho_d$ such that $\sum_{k=1}^d \rho_k = 1$, $(c_1, \ldots, c_d)$ the vector of constant premium rates, $\{(X_{1i}, \ldots, X_{di}); i \geq 1\}$ the sequence of claim size vectors, and $\tau_1, \tau_2, \ldots$ the common claim-arrival times which constitute a renewal claim-number process $\{N(t); t \geq 0\}$ with finite renewal function $\lambda(t) = \mathbb{E}N(t) = \sum_{i=1}^\infty \mathbb{P}(\tau_i \leq t)$.

Hereafter, for the conciseness in expression, we shall mainly formulate our problems in a vector notation. Unless otherwise

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stated, a vector will be written as a bold letter such as $\mathbf{a}$ and will be assumed to be d-dimensional. Particularly, we use a bold Arabic number to stand for the vector with all components being that number, e.g., $\mathbf{0} = (0, \ldots, 0)$ and $\mathbf{1} = (1, \ldots, 1)$. For two vectors $\mathbf{a}$ and $\mathbf{b}$ of the same dimension, we understand relations such as $\mathbf{a} \geq \mathbf{b}$ and $\mathbf{a} \pm \mathbf{b}$ as componentwise. Additionally, the scalar multiplication is defined as usual, i.e., $y\mathbf{a} = (ya_1, \ldots, ya_d)$ with y being a real number. With the above conventions, the d-dimensional risk model (1.1) can be rewritten as

$$\mathbf{U}(t) = \rho x e^{R(t)} + \mathbf{c} \int_0^t e^{R(t-s)} ds - \sum_{i=1}^{N(t)} \mathbf{X}_i e^{R(t-\tau_i)}, \quad t \geq 0.$$

Unlike the one-dimensional case, there are many optional ways to define the ruin time for a multi-dimensional risk model; see Li et al. (2007) or Chen et al. (2011) for some commonly used ones. In this paper, we consider the following two types of ruin times defined as

$$T_{\max} = \inf\left\{ s > 0 : \max_{1\leq k\leq d} \{U_k(s)\} < 0 \right\} = \inf\{s > 0 : \mathbf{U}(s) < \mathbf{0}\}, \tag{1.2}$$

which is the first moment when all the businesses go into deficit, and

$$T_{\min} = \inf\left\{ s > 0 : \min_{1\leq k\leq d} \{U_k(s)\} < 0 \right\}, \tag{1.3}$$

which is the first moment when there is some business going into deficit. Here, $\inf \emptyset = \infty$ by convention. Correspondingly, the two types of ruin probabilities of risk model (1.1) can be written as

$$\psi_* (x; t) = \mathbb{P}(T_* \leq t \,|\, \mathbf{U}(0) = \rho x), \quad 0 < t \leq \infty, \tag{1.4}$$

where $T_* \leq t$ is understood as $T_* < \infty$ if $t = \infty$ and "$*$" may be one of "max" and "min". Note that $\psi_* (x; t)$ with finite t is usually called the finite-time ruin probability, while $\psi_* (x; \infty)$ the infinite-time ruin probability. In this paper, we aim to derive the precise asymptotic expansions for $\psi_* (x; t)$ as $x \to \infty$ with the uniformity for the whole time horizon.

In recent years, the study on the asymptotic behavior of ruin probabilities of various multi-dimensional risk models has already become a hot topic in the field of risk theory, since the classic one-dimensional models obviously cannot provide the whole picture for effects of different businesses to an insurer's solvency. However, subject to mathematical tractability, most works of this topic are concentrated on typical but special two-dimensional models with independence assumptions among claim sizes from different businesses or that between claim sizes and claim-number processes; see Li et al. (2007), Chen et al. (2011), Zhang and Wang (2012), Chen et al. (2013a,b), Hu and Jiang (2013), Yang and Li (2014), and Li (in press), among many others.

Compared to fruitful results on the study for two-dimensional risk models, the investigation for the corresponding higher-dimensional models is quite limited. Among the few contributions, Huang et al. (2014) considered a discrete-time multi-dimensional risk model, in which the claim sizes from different businesses follow a dependence structure given in terms of copulas. Under the assumption that the claim sizes have regularly varying tails, the authors obtained asymptotic expansions for finite-time ruin probabilities. Additionally, Li et al. (2015) studied a continuous-time multi-dimensional risk model, which is a special case of (1.1) with $R(t) \equiv 0$ (i.e., interest-free) and $\{N(t); t \geq 0\}$ being a Poisson process. They still only focused on finite-time ruin probabilities and obtained the corresponding asymptotic formulas for asymptotically independent regularly varying claim sizes. The most recent contributions on the topic are Samorodnitsky and Sun (2016) and Konstantinides and Li (2016). The former contribution successfully introduced a concept of multivariate subexponentiality and applied it in a interest-free multi-dimensional risk model with independence assumption between the claim sizes and their common claim number process. The latter work proposed a general multi-dimensional risk model, in which the claim-number processes of different businesses may not be same and the claim sizes were modeled via a framework of multivariate regular variation. Assuming that the claim sizes and their claim-number processes are independent and that the interest rate is constant (i.e., $R(t) = rt$ for some $r > 0$), Konstantinides and Li (2016) obtained asymptotic expansions for $\psi_{\max} (x; t)$ for fixed $0 < t \leq \infty$.

The current paper extends the existing works on multi-dimensional risk models mainly from three aspects. Firstly, we take into account both the dependence among the claim sizes and that between the claim sizes and their common claim-number process; see Assumption 2.2. Secondly, we equip our risk model with the stochastic accumulation factor $\{e^{R(t)}; t \geq 0\}$ which covers the traditional setup of constant interest rate. Thirdly, the obtained asymptotic formulas for $\psi_* (x; t)$ possess the uniformity for the whole time horizon.

In the rest of this paper, Section 2 states our main result after introducing necessary preliminaries, including basic properties of regular varying distribution functions and the dependence assumptions introduced to the risk model, Section 3 gives a specific example for which our dependence assumptions are satisfied, and Section 4 proves the main result after preparing some useful lemmas.

2. Preliminaries and main results

Hereafter, all limit relations hold as $x \to \infty$ unless otherwise stated. For two positive functions f and g, we write $f(x) \lesssim g(x)$ or $g(x) \gtrsim f(x)$ if $\limsup f(x)/g(x) \leq 1$ and write $f(x) \sim g(x)$ if both $f(x) \lesssim g(x)$ and $f(x) \gtrsim g(x)$. For two real numbers a and b, we write $a \wedge b = \min\{a, b\}$ and write $a \vee b = \max\{a, b\}$.

2.1. Regular variation

A distribution function $F = 1 - \overline{F}$ on $[0, \infty)$ is said to be of regular variation if $\overline{F}(x) > 0$ for all $x \geq 0$ and the relation

$$\lim_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = y^{-\alpha}, \quad y > 0, \tag{2.1}$$

holds for some $0 \leq \alpha < \infty$. We signify the regularity property in (2.1) as $F \in \mathcal{R}_{-\alpha}$.

For a distribution function $F \in \mathcal{R}_{-\alpha}$ for some $0 < \alpha < \infty$, by Proposition 2.2.3 of Bingham et al. (1987) we know that, for any $0 < \varepsilon < \alpha$ and $A > 1$, there is some $x_0 > 0$ such that the inequalities

$$\frac{1}{A}\left(y^{-\alpha+\varepsilon} \wedge y^{-\alpha-\varepsilon}\right) \leq \frac{\overline{F}(xy)}{\overline{F}(x)} \leq A\left(y^{-\alpha+\varepsilon} \vee y^{-\alpha-\varepsilon}\right) \tag{2.2}$$

hold whenever $x > x_0$ and $xy > x_0$. We can derive from (2.2) that if $\overline{F} \in \mathcal{R}_{-\alpha}$ then, for any $p > \alpha$,

$$x^{-p} = o\left(\overline{F}(x)\right), \quad x \to \infty. \tag{2.3}$$

We refer the reader to Bingham et al. (1987) and Embrechts et al. (1997) for more details on the distribution class of regular variation.

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