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The role of the dependence between mortality and interest rates when pricing Guaranteed Annuity Options

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ABSTRACT

In this paper we investigate the consequences on the pricing of insurance contingent claims when we relax the typical independence assumption made in the actuarial literature between mortality risk and interest rate risk. Starting from the Gaussian approach of Liu et al. (2014), we consider some multifactor models for the mortality and interest rates based on more general affine models which remain positive and we derive pricing formulas for insurance contracts like Guaranteed Annuity Options (GAOs). In a Wishart affine model, which allows for a non-trivial dependence between the mortality and the interest rates, we go far beyond the results found in the Gaussian case by Liu et al. (2014), where the value of these insurance contracts can be explained only in terms of the initial pairwise linear correlation.

1. Introduction

A large number of life insurance products, such as annuities, include interest rate and mortality risks. Mortality risk is generally considered to be less difficult to be modeled than interest risk. Indeed, by virtue of the law of large numbers, actuarial practice considered for a long time that the mortality risk can be diversified away by holding a sufficiently large portfolio of similar contracts (see e.g. Milevsky and Young, 2007 for a discussion). Therefore, the traditional approach of actuaries consisted in modeling mortality in a deterministic way as opposed to interest rates which were assumed stochastic for quite some time now. Later, when stochastic mortality models showed up during the 90s, the actuarial community made the assumption that mortality risk is independent of interest risk. This assumption may seem acceptable in the short term. Nevertheless, it seems also natural that catastrophic risks, like an earthquake or a severe pandemic, will affect the economy and the financial markets in the short run. Furthermore, in the long term, it seems intuitive that demographic changes can affect the economy. For example, in Favero et al. (2011), the authors investigate the possibility that the slowly evolving mean in the log dividend–price ratio is related to demographic trends. Maurer (2014) explores how demographic changes affect the value of financial assets. He considers a continuous time overlapping generations model where birth and mortality rates are stochastic. His model suggests that demographic transitions explain substantial parts of the time variation in the real interest rate, equity premium and conditional stock price volatility. Moreover, he gives sufficient conditions for the interest rate to be decreasing in the birth rate and increasing in the death rate. In Dacorogna and Cadena (2015), the authors are interested in providing some empirical evidence of a changing behavior of the economy and the financial markets during periods where the mortality is relatively high. Another motivation of dependence between the mortality and interest rates can be found in Nicolini (2004) where it is shown that the increase in adult life expectancy in the 17th and the 18th century can be considered a key factor in explaining the increase in the accumulation of assets and the decline in the interest rate that took place in pre-industrial England. To conclude, we mention that Dhaene et al. (2013) investigate the conditions under which it is possible (or not) to transfer the independence assumption from the physical world to the pricing world. In particular, they show that this independence relation is not maintained in general. Therefore, as also

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suggested by Miltersen and Persson (2005) and Liu et al. (2014), it is more reasonable to have a pricing framework that allows for a dependence between mortality and interest rates.

Nowadays, it is widely admitted that mortality intensities behave in a stochastic way. There are important similarities between the force of mortality and interest rates (see e.g. Milevsky and Promislow, 2001; Dahl, 2004; and Biffis, 2005). Luciano and Vigna (2008) calibrate some affine models where the force of mortality follows either a Cox–Ingersoll–Ross (CIR) process or an Ornstein–Uhlenbeck process to different generations in the UK population and investigate their empirical relevance. They find that Ornstein–Uhlenbeck processes (with zero long-term mean) seem to be appropriate descriptors of human mortality. Dahl (2004), Dahl and Moller (2006) and Dahl et al. (2008) treat the classical affine situation for both stochastic mortality rates and interest rates, but in the independent situation under both the historical and the real world measure. Russo et al. (2011) focus upon the calibration of affine stochastic mortality models using term insurance premiums.

In this paper we assume that the interest rate and the mortality dynamics are not independent of each other. More precisely, we consider a general affine framework like in Keller-Ressel and Mayerhofer (2015); our goal is to study the influence in pricing of their dependence structure when focusing upon mortality and interest rates in different models constructed by a linear combination of positive idiosyncratic and systematic processes, inspired by factor models as in Duffie and Kan (1996) and Duffie and Garleanu (2001). These linear combinations can be chosen so that the model either avoids or allows interest rates and/or mortality rates to be negative. For mortality rates, it is a desirable property to remain positive. However, the financial markets show the recent years negative interest rates, see e.g. Borovkova (2016), Recchioni and Sun (2016) or Russo and Fabozzi (2016). Although the spreads between the LIBOR rates and the overnight indexed swap (OIS) rates of the same maturity have been far from negligible since 2007, and several multiple curve interest rate models have been introduced (see e.g. Grbac and Runggaldier, 2015), we choose in the present paper the interest rate models still from the traditional single-curve models in order to study the influence of the dependence between interest rates and mortality rates.

In particular, generalizing the investigations of Liu et al. (2014), we are interested in two multifactor specifications that are nested in the general affine framework, namely the multi-CIR and the Wishart model that have been successfully applied to many fields of quantitative finance. Wishart processes have been first defined by Bru (1991) and recently introduced in finance by Gouriéroux and Sufana (2003, 2011). They represent a matrix extension of the square-root model that allows for a non trivial correlation between the diagonal terms which are by definition positive (see e.g. Cuchiero et al., 2011 for a complete characterization). This property enables to overcome an intrinsic constraint of the standard affine (Duffie and Kan, 1996) model. In addition, the affine property of the Wishart process leads to a closed form expression for its moment-generating function, so that pricing within the Wishart framework can be efficiently performed via Fourier methods.[1]

From the empirical side, the advantages of interest rate models based on the Wishart process have been underlined by Buraschi et al. (2008) and Chiarella et al. (2016). The latter extends the former by estimating, using Kalman filtering techniques, an extended version of the classical Wishart model, together with a multifactor CIR model. The Wishart based model was found to outperform the multifactor CIR both in terms of goodness of fit and hedging performance.

To the best of our knowledge, the first attempt to introduce dependence between mortality and interest rates, was done by Miltersen and Persson (2005), see also Cairns et al. (2006). We follow the methodology of Jalen and Mamon (2009) who introduced a pricing framework in which the dependence between the mortality and the interest rates is explicitly modeled. We apply the same change of probability measure to the valuation of some insurance contracts such as indexed annuities and Guaranteed Annuity Options (GAOs). These options are available to holders of certain life insurance policies and give them the right to convert their accumulated funds to a life annuity at a fixed rate when the policy matures. This kind of guarantees became very popular in the 1970s and 1980s when long term rates in many countries were quite high. We mention that in Liu et al. (2014), a pricing formula for the GAO has been obtained where the interest rate and mortality processes follow bivariate Gaussian dynamics. In their setting, the dependence between mortality and interest rates is described just by one constant, namely the pairwise linear correlation coefficient. We will show that in (positive) structures such as the Wishart framework the aforementioned dependence shows richer features.

The remainder of this paper is organized as follows. In Section 2, we present our model in a general affine approach and we define the change of probability measure used in the sequel for the pricing purpose. In Section 3, we concentrate upon some insurance products and we present different ways for determining their fair values in our setting. In Section 4, we specify the dynamics of the affine processes in two important specifications: the multidimensional CIR process and the Wishart case. For both settings we derive formulas for the price of the insurance contracts of Section 3. In Section 5 we perform a sensitivity analysis with respect to parameters that rule the dependence structure between interest rates and mortality risks. We provide concluding remarks in Section 6. Finally, we gather in the Appendix some technical results and a brief overview on affine processes.

2. The general pricing model

We consider a filtered probability space $(\Omega, \mathcal{G}, (\mathcal{G}_t)_t, \mathbb{Q})$ where $\mathbb{Q}$ is a risk-neutral martingale measure. By the presence of both mortality and interest rate risk, we are dealing with a pricing problem in an incomplete market. Following the standard practice, we assume that the right risk-neutral probability can be selected on the basis of e.g. available market data (for another approach about the non-diversifiable mortality risk in an incomplete market, see e.g. Bayraktar et al., 2009).

We denote by $\tau_M(x)$ the positive random variable corresponding to the future lifetime of an individual aged x at time 0, admitting a random intensity $\mu(s, x+s)$. As in e.g. Biffis (2005), we regard $\tau_M(x)$ as the first jump-time of a nonexplosive $\mathcal{G}$-counting process N recording at each time $t \geq 0$ whether the individual died ($N_t \neq 0$) or not ($N_t = 0$). Let $\mathcal{R}_t$ be the filtration generated by the interest rate process and $\mathcal{M}_t$ the one associated to the mortality intensity. We denote by $\mathcal{F}_t := \mathcal{R}_t \vee \mathcal{M}_t$ the minimal σ-algebra containing $\mathcal{R}_t \cup \mathcal{M}_t$. The filtration $(\mathcal{G}_t)_t$ denotes the flow of information available as time goes by: this includes knowledge of the evolution of both state variables above up to each time t and of whether the policyholder has died by then. Therefore, $\mathcal{G}_t := \mathcal{F}_t \vee \mathcal{H}_t$, with

$$\mathcal{H}_t := \sigma(1_{\{\tau_M(x) \leq s; 0 \leq s \leq t\}}),$$

[1] The Wishart process has found applications to many fields of quantitative finance like multivariate option pricing (see e.g. Da Fonseca et al., 2007, 2008; Da Fonseca and Grasselli, 2011), yield curve modeling (see e.g. Buraschi et al., 2008; Gnoatto, 2012; Chiarella et al., 2014; Da Fonseca et al., 2013), credit risk (Gouriéroux and Sufana, 2010), portfolio management (Buraschi et al., 2010; Da Fonseca et al., 2011), commodity derivative pricing (Chiu et al., 2015) and foreign exchange models (Leung et al., 2013; Branger and Muck, 2012; Gnoatto and Grasselli, 2014).