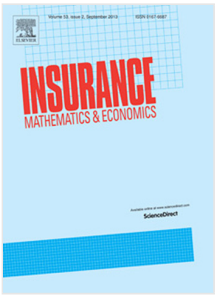

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Constrained investment-reinsurance optimization with regime switching under variance premium principle

Lv Chen[a], Linyi Qian[a], Yang Shen[b,*], Wei Wang[c]

[a]*School of Statistics, East China Normal University, 500 Dongchuan Road, Shanghai 200241, China*
[b]*Department of Mathematics and Statistics, York University, Toronto, Ontario, M3J 1P3, Canada*
[c]*Department of Financial Engineering, Ningbo University, 818 Fenghua Road, Ningbo 315211, China*

Abstract

This paper studies optimal investment and reinsurance problems for an insurer under regime-switching models. Two types of risk models are considered, the first being a Markov-modulated diffusion approximation risk model and the second being a Markov-modulated classical risk model. The insurer can invest in a risk-free bond and a risky asset, where the underlying models for investment assets are modulated by a continuous-time, finite-state, observable Markov chain. The insurer can also purchase proportional reinsurance to reduce the exposure to insurance risk. The variance principle is adopted to calculate the reinsurance premium, and Markov-modulated constraints on both investment and reinsurance strategies are considered. Explicit expressions for the optimal strategies and value functions are derived by solving the corresponding regime-switching Hamilton-Jacobi-Bellman equations. Numerical examples for optimal solutions in the Markov-modulated diffusion approximation model are provided to illustrate our results.

Keywords: Investment; Reinsurance; Regime switching; Variance premium principle; Hamilton-Jacobi-Bellman equation

1. Introduction

Mixed operation of financial institutions has witnessed that insurance companies are playing an increasingly important role in the whole finance industry, and that other financial institutions tend to involve themselves in the insurance industry. The traditional business of insurance companies is to undertake insurance risk by providing protection to policyholders. Doing so, insurance companies receive insurance premium from a large number of policyholders. Due to the limitation of capital and the regulatory restrictions, insurance companies usually cede part of insurance risk to reinsurance companies by sacrificing and transferring part of insurance premium. The transferred insurance premium is called reinsurance premium. In practice, the reinsurance premium rate is usually higher than the insurance premium rate, that is, the reinsurance is non-cheap. Thus, it is relevant to consider the trade-off between risk and return from the perspective of insurance companies. The problem of determining how much insurance risk an insurance company should cede or retain is termed as an optimal reinsurance problem. On the other hand, insurance companies have been active and major participants in secondary markets nowadays. The investment activity allows insurance companies to generate profits so as to hedge against insurance risk and achieve other financial goals, such as maximizing the expected utility, minimizing the ruin probability and optimizing the mean-variance criterion. As a result, an optimal investment problem naturally arises in insurance companies' agenda. This motives recent study on optimal investment and reinsurance rules from various perspectives in actuarial science.

Indeed, over the past decade or two, there have been dozens of works on optimal investment and reinsurance problems. Browne (1995) used a drifted Brownian motion to model the surplus process and studied the problems of minimizing the risk of ruin and maximizing the expected utility of terminal wealth. Yang and Zhang (2005) considered optimal investment policies for an insurer, whose surplus follows jump-diffusion processes and that

*Corresponding author.
Email addresses: chenlvhero@sina.com (Lv Chen), lyqian@stat.ecnu.edu.cn (Linyi Qian), skyshen87@gmail.com (Yang Shen), wswang2008@163.com (Wei Wang)