Insurance: Mathematics and Economics 71 (2016) 353–366

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Demand for longevity securities under relative performance concerns: Stochastic differential games with cointegration[☆]

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ARTICLE INFO

Article history:
Received July 2016
Received in revised form
September 2016
Accepted 9 October 2016
Available online 21 October 2016

Keywords:
Nonzero sum games
Longevity security market
Cointegration
Relative performance
Nash equilibrium
Demand of longevity bonds

ABSTRACT

This paper investigates the impact of relative performance concerns on the longevity risk transfer market. When an insurer concerns about the relative performance in a two-insurer economy, she maximizes the expected utility of her terminal wealth benchmarked against her competitor's. The problem formulation for a general utility, a general interest rate process and cointegrated mortality rates uses a nonzero sum stochastic differential game approach. Explicit solution of the Nash equilibrium is derived for constant relative risk adverse insurers under the Vasicek-type stochastic interest and mortality rates. Existence and uniqueness of the Nash equilibrium are established for the CIR-type models, which rule out negative interest and mortality rates. While previous studies based on the single-agent approaches have shown a high investment demand in longevity bonds, the launch of it was unsuccessful in reality. Ours supplements that the demand is much lower subject to the relative performance concerns.

1. Introduction

While the prolonged life expectancy is beneficial for individuals and the society, it is the financial risk for the government, insurers and pension funds. Longevity market is needed to transfer and share the risk with the public as well as creating profits and diversification opportunities. The International Monetary Fund (IMF, 2012) highly recommends the development a liquid longevity risk transfer market to strengthen the stability of sovereign balance sheets. Investment banks have been interested in making the longevity market and LifeMetrics is a typical quantitative toolkit developed by JP Morgan to measuring and managing longevity risk (Loeys et al., 2007).

Longevity securitization includes structuring mortality-based securities such as the survivor swaps (Dowd et al., 2006) and the longevity bonds (Blake et al., 2006). For example, a zero-coupon longevity bond pays the holder the face value times the percentage of survivors in a population upon maturity. It offers a yield higher than the risk-free interest rate to compensate for the mortality risk. Assuming a liquid longevity market, theoretical results have been developed for longevity risk management with longevity securities such as Menoncin (2008), Cairns et al. (2014), Wong et al. (2014), Wong et al. (2015) and Biagini et al. (2015).

While the longevity risk transfer market is important for pension providers to hedge against the longevity risk, is it beneficial to other investors? A liquid market is a consequence of trading incentives of both purchasing and selling the securities in the market. The literature addresses this issue through the optimal consumption and investment problems with mortality contingent claims. Farhi and Panageas (2007), and Dybvig and Liu (2010) study the optimal consumption and portfolio choice for a finite-lived representative agent using a stochastic optimal control framework. By adding longevity securities into the analysis, a significant demand of longevity bonds to hedge against the shocks of life expectancy is shown in Cocco and Gomes (2012) using a representative agent approach. By calibrating to the US females population, the optimal consumption/investment model in Menoncin and Regis (2015) further confirms that the demand for longevity bonds is significant for this population even after controlling the sensitivities of risk aversion and other parameters. Maurer et al. (2013) assess the importance of variable annuities in smoothing consumption.

In reality, the launch of longevity securities is much less successful than theoretical models predict. In 2004, the European Investment Bank announced plans to issue the first longevity bond but it was withdrawn in late 2005 without being issued, primarily

☆ M.C. Chiu acknowledges the support by the Research Grant Council of Hong Kong with GRF Project Number: 18200114.

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http://dx.doi.org/10.1016/j.insmatheco.2016.10.005
0167-6687/

because the pension industry perceived the price of coverage on longevity risk too high. Blake et al. (2013) suggest some important ingredients for a liquid life market, including the optimal contract design. A consultative document by the Bank for International Settlements (BIS) in BIS Joint Forum (2013) addresses obstacles and potentials for the development of the longevity markets.

The classical analysis of the demand on longevity securities and/or optimal contract design stems on the single representative agent's decision for the entire economy. However, financial institutions primarily concern with their performances relative to their competitors (DeMarzo et al., 2008). The corresponding optimal equilibrium decision is closely related to financial bubbles, which the approach of representative agent fails to address. A tractable framework is proposed in Espinosa and Touzi (2015) to model the interaction among heterogeneous agents under the Brownian motion framework. Applying this concept to a two-person game in insurance, Bensoussan et al. (2014) investigate the optimal investment–reinsurance decision by formulating the problem as a nonzero sum stochastic differential game. Pun et al. (2016) generalize their result to the case of ambiguous correlation using the framework of Fouque et al. (2016). The present paper explores the optimal longevity investment under the relative performance concerns and examines the implication for the demand of longevity bonds.

Specifically, we consider two insurers who aim to optimally allocate their wealth among a longevity bond, a risk-free bond and the bank account. Each insurer is subject to a random insurance liability following a doubly stochastic Poisson process, with an intensity rate being correlated and cointegrated with the index mortality rate underlying the longevity bond. Each of the two insurers wishes to beat the other by maximizing the expected utility on a relative wealth. This two-person optimization problem leads to a nonzero sum stochastic differential game. Their optimal investment is the Nash equilibrium of the game upon existence of the solution.

We first characterize the problem under the general theoretical setup. Specific results are then derived for two different market conditions: Deterministic market price of risk with Vasicek setting, and volatility driven market price of risk with the extended CIR setting. Both are considered in Wong et al. (2015). Under the first market condition, we derive the explicit solution of the Nash equilibrium strategies and the optimal objective function for the constant relative risk adverse (CRRA) insurers. The explicit solution enables us to examine insurers' equilibrium demand on longevity bonds under relative performance concerns. Under the second market condition, explicit solution is not available so that we prove for the existence and uniqueness of the Nash equilibrium and devise a numerical implementation in our numerical demonstration section.

Typically, individual insurer's demand on longevity bond increases with the degree of cointegration between her mortality exposure and the mortality index. It articulates the important role of cointegration in longevity security demand. However, the insurer decreases her demand on longevity security when her competitor's portfolio is more cointegrated with the mortality index. In other words, when the longevity market is in favor of the risk management of the competitor's portfolio through hedging, the underlying insurer is intended to lower her participation in the longevity market, and eventually reduces its liquidity, to maintain her industrial ranking.

The contribution of this paper is twofold. Mathematically, we formulate the optimal longevity investment decision under the relative performance concerns as nonzero sum stochastic differential games, and solve explicit solutions to some meaningful model settings. Specifically, the Nash equilibrium is shown to be the solution of a pair of Hamilton–Jacobi–Bellman (HJB) equations associated with a jump–diffusion model, where jumps are caused by the risk processes of insurers. The unique Nash equilibrium is obtained as a pair of explicit formulas for CRRA insurers under the Vasicek setting. We prove existence and uniqueness for the Nash equilibrium under the CIR setting.

Financially, our result offers new insights into the impact of competition between insurers on the longevity market. While the representative-agent approach suggests a significant investment demand on longevity bonds, the actual demand could be much less once insurers aim to beat their competitors. Consequently, insurers reduce their demands on longevity bonds for maintaining their industrial ranking once their core business is highly dependent with the longevity bonds. This suggests that the optimal longevity security design, which takes into account of relative performance concerns, possess a very interesting future research.

The remainder of this paper is organized as follows. Section 2 presents the model and the framework of the nonzero sum stochastic differential game. The solution process of the Nash equilibrium is detailed in Section 4. Section 5 characterizes the Nash equilibrium to some meaningful model settings. Section 6 provides the numerical examples of the two-person nonzero sum game and draws financial interpretations from the model parameters. Section 7 concludes the paper.

2. Problem formulation

This section begins with the interest rate and mortality rate model with cointegration, which are the building block for longevity bond pricing and the wealth processes of insurers. We then present the problem formulation of the optimal longevity investment for insurers under the relative performance concerns.

2.1. The stochastic model

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete filtered probability space and $[0, T_0]$ be the planing horizon, with $0 < T_0 < T < \infty$, with T be the maturity of the risk-free interest rate and zero coupon longevity bonds. $\mathcal{F}_t$ is the σ-field generated by the n-dimensional standard Brownian motion $\{W_i(t)\}_{i=1}^n$ under $\mathbb{P}$. In our problem, it involves a stochastic interest rate and at least three stochastic mortality rates, therefore $n \geq 4$. Specially, $W(t) = (W_r(t), W_{\lambda_0}(t), W_{\lambda_1}(t), W_{\lambda_2}(t))'$ is a vector of independent Wiener processes under $\mathbb{P}$.

Consider an economy of two insurers who aim to optimally allocate their wealths among a bank account, a zero-coupon risk-free bond and a zero-coupon longevity bond. The $\mathcal{F}_t$-adapted risk-free interest rate is the rate of return of the bank deposit such that $dB(t) = r(t)B(t)dt$ and $B(0) = 1$, where $B(t)$ is the bank deposit amount at time t. Hence, $B(t) = \exp\left(\int_0^t r(s)ds\right)$. Although a more general model for the interest rate is possible, we use the following stochastic differential equation (SDE) for the interest rate dynamics.

$$dr(t) = (\beta_r - \alpha_r r(t))dt + \sigma_r(t, r(t))dW(t), \tag{1}$$

with deterministic β_r and α_r, and a $\mathbb{R}^4$-valued function $\sigma_r(t, r(t))'$.

The stochastic interest rate makes the zero coupon bond different from the bank account. A zero coupon bond paying \$1 at maturity T reads

$$\begin{aligned}\mathcal{B}(t,T) &= \mathbb{E}^{\mathbb{P}}\left[\exp\left(-\int_t^T r(s) + \mu_{\mathcal{B}}(s)ds\right) \,\Big|\, \mathcal{F}_t\right] \\ &= \mathbb{E}^{\mathbb{Q}}\left[\exp\left(-\int_t^T r(s)ds\right) \,\Big|\, \mathcal{F}_t\right],\end{aligned} \tag{2}$$

where $\mathbb{Q}$ is the risk-neutral measure and $\mu_{\mathcal{B}}(t)$ is the $\mathcal{F}_t$-adapted excess rate of return implied by the interest rate term structure.