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Impact of volatility clustering on equity indexed annuities

Donatien Hainaut

ISBA, UCL, Voie du Roman Pays 20, B-1348, Louvain-la-Neuve, Belgium

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ABSTRACT

This study analyses the impact of volatility clustering in stock markets on the evaluation and risk management of equity indexed annuities (EIA). To introduce clustering in equity returns, the reference index is modelled by a diffusion combined with a bivariate self-excited jump process. We infer a semi-closed form or parametric expression of the moment generating functions in this framework for the equity return and the intensities of jumps. An econometric procedure is proposed to fit the model to a time series. Next, we develop a method, based on a normal inverse Gaussian approximation of the index return, to evaluate options embedded in simple variable annuities. To conclude, we compare prices, one-year value at risks, and tail value at risks of simple EIAs, computed with different models.

1. Introduction

Significant volumes of equity indexed annuities (EIA) are sold in the US insurance market. They provide a guaranteed annual return combined with some participation in equity market appreciation. Participation in an equity index is offered by guaranteeing a specified proportion (the participation rate) of the return on an index, with a floor and a cap. The valuation of options embedded in EIAs and the wide variety of EIAs is at the origin of abundant literature. Bacinello (2003) and Bauer et al. (2008) proposed a unified framework to evaluate variable annuities. Milevski and Salisbury (2006), and Dai et al. (2008), studied the valuation of guaranteed minimum withdrawal benefits. Hardy (2003) presented an overview of various investment guarantees. Much attention has been given to the EIAs valuation under the Black–Scholes framework, including studies by Tiong (2000), Lee (2003), and Lin and Tan (2007). Recently, researchers investigated the influence of alternative dynamics on EIA prices. For example, Gerber et al. (2013) appraised equity-linked death benefits in a jump diffusion setting. Siu et al. (2014) and Fan et al. (2015) evaluated EIA when the reference index was a switching Brownian motion. Kelani and Quittard Pinon (2014), and Balotta (2009) studied ratchet options when the underlying asset is led by Lévy processes.

Jumps or Lévy processes capture stylized features of stock market dynamics like negative skewness and an excess of kurtosis. However, they have stationary increments and thus do not exhibit any clustering of large jumps. On the other hand, empirical analysis conducted in Ait-Sahalia et al. (2015) and Embrechts et al. (2011) emphasizes the importance of this effect in financial markets. Clustering is not characterized by a single jump but by the amplification of movement that takes place subsequently over several days. However, to the best of our knowledge, most of the models commonly used for the pricing of EIAs do not integrate this characteristic. This observation motivates the developments proposed in this work.

We model the EIA index by a jump diffusion combined with a mechanism of mutual excitation between positive and negative shocks to introduce clustering of volatility in equity returns. Such an approach was introduced in the econometric literature by Ait-Sahalia et al. (2015), Giot (2005), Chavez-Demoulin et al. (2005), and Chavez-Demoulin and McGill (2012), and is based on Hawkes processes (see Hawkes, 1971a,b or Hawkes and Oakes, 1974). These are parsimonious self-exciting point processes for which the intensity jumps in response to a shock and reverts to a target level in the absence of an event. Hawkes processes are increasingly integrated in high frequency finance. Examples include modelling the duration between trades (Bauwens and Hautsch, 2009), and the arrival process of buy and sell orders, as in Bacry et al. (2013).

This study contributes to the literature on variable annuities in several ways. Firstly, the proposed model allows for volatility clustering, a feature that is absent from most of the previous studies on EIAs. Secondly, we find semi-closed form expressions for the moment generating function (MGF) of the index, and of the intensities of jump processes. When the positive and negative jumps are self-excited but mutually independent, we show that these MGFs admit parametric closed form expressions. Thirdly, an econometric method to fit the model to the time series is

E-mail address: donatien.hainaut@uclouvain.be.

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developed. This study also introduces a method to evaluate options embedded in simple variable annuities based on a normal inverse Gaussian approximation of the index return. Finally, we conduct a complete numerical illustration in which prices and risk of EIA linked to the S&P 500 are assessed. The results are then compared to those obtained with pure jump diffusion and lognormal models.

The rest of the paper is organized as follows. Section 2 presents the dynamics of the reference index and its characteristics. Section 3 details the econometric procedure to estimate parameters from a time series. Section 4 focuses on the pricing of EIAs. Section 5 is a numerical application in which we quantify the impact of volatility clustering on prices and risk of EIAs.

2. The mutually excited jumps diffusion model (MEJD)

EIAs are saving instruments that provide some participation in equity market appreciation. This section describes the dynamics of the stock index that serves as a reference for the determination of the EIA participation rate. The mechanism of the EIA is detailed later in Section 4. We consider a complete probability space $(\Omega, \mathcal{F}, P)$ that is equipped with the filtration $\mathcal{F}_t$, generated by an equity index, S_t. The dynamics of S_t is defined by the following stochastic differential equation SDE:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma dW_t + (e^{J_1} - 1)dN_t^1 + (e^{J_2} - 1)dN_t^2 \tag{1}$$

where W_t is a Brownian motion and σ is the related constant volatility. N_t^1 and N_t^2 are two independent point processes with intensities denoted by λ_t^1 and λ_t^2. J^1 and J^2 are positive and negative exponential Jumps, respectively. The drift of S_t is such that the expected instantaneous return is constant and equal to μ:

$$\mu_t = \mu - \lambda_t^1 \mathbb{E}(e^{J_1} - 1) - \lambda_t^2 \mathbb{E}(e^{J_2} - 1). \tag{2}$$

The densities of exponential jumps, denoted by $(\nu_i(z))_{i=1,2}$, are defined by two parameters $\rho_1,\ \rho_2 \in \mathbb{R}^+$ as follows:

$$\nu_1(z) = \rho_1 e^{-\rho_1 z} 1_{\{z\geq 0\}}, \tag{3}$$
$$\nu_2(z) = \rho_2 e^{+\rho_2 z} 1_{\{z<0\}}.$$

The average sizes of jumps $(J_i)_{i=1,2}$ are equal to $\mathbb{E}(J_1) = \frac{1}{\rho_1}$ and $\mathbb{E}(J_2) = -\frac{1}{\rho_2}$. In the following developments, the moment generating function of these jumps is noted:

$$\psi^1(z) := \mathbb{E}\left(e^{zJ_1}\right) = \frac{\rho_1}{\rho_1 - z} \quad z < \rho_1$$
$$\psi^2(z) := \mathbb{E}\left(e^{zJ_2}\right) = \frac{\rho_2}{\rho_2 + z} \quad -\rho_2 < z.$$

The clustering of shocks is modelled by the assumption that the frequencies depend on the jumps themselves:

$$d\lambda_t^1 = \kappa_1(c_1 - \lambda_t^1)dt + \delta_{1,1}J_1 dN_t^1 + \delta_{1,2}J_2 dN_t^2 \tag{4}$$
$$d\lambda_t^2 = \kappa_2(c_2 - \lambda_t^2)dt + \delta_{2,1}J_1 dN_t^1 + \delta_{2,2}J_2 dN_t^2$$

and revert to a constant level $c_{1,2}$ at a speed $k_{1,2}$. The constraints, $\delta_{1,1},\ \delta_{2,1} > 0$ and $\delta_{2,2},\ \delta_{1,2} < 0$, are added to ensure the positivity of λ_t^1 and λ_t^2. We call this model the mutually excited jumps diffusion model (MEJD) in the remainder of the article. Eqs. (4) ensure the presence of contagion between positive and negative jumps.

The intensities are assumed to be observable. This assumption is not penalizing because jumps are detectable by a "peak over threshold" method and the paths of λ_t^1 and λ_t^2 can be retrieved by a log likelihood maximization. These points are detailed and illustrated in Section 3. In the next developments, the infinitesimal generator of S_t of a real function $f(t, S_t, \lambda_t^1, \lambda_t^2)$ is defined by the following expression:

$$\begin{aligned}
\mathcal{A}f(t, S_t, \lambda_t) &= f_t + \mu S_t f_S + \frac{1}{2}\sigma^2 S_t^2 f_{SS} + \sum_{i=1,2} \kappa_i(c_i - \lambda_t^i) f_{\lambda^i} \\
&+ \lambda_t^1 \int_{-\infty}^{-\infty} f\left(t,\ S_t + S_t\left(e^z - 1\right),\ \lambda_t^1 + \delta_{1,1}J_1,\ \lambda_t^2 + \delta_{2,1}J_1\right) \\
&- f - (e^z - 1)S_t f_S \nu_1(dz) \\
&+ \lambda_t^2 \int_{-\infty}^{-\infty} f\left(t,\ S_t + S_t\left(e^z - 1\right),\ \lambda_t^1 + \delta_{1,2}J_2,\ \lambda_t^2 + \delta_{2,2}J_2\right) \\
&- f - (e^z - 1)S_t f_S \nu_2(dz)
\end{aligned} \tag{5}$$

where f_t, f_S, f_{SS}, and f_{λ_i} are partial derivatives with respect to time, to S_t, and to $\lambda_t^{1,2}$, respectively. On the other hand, the infinitesimal variation of f is ruled by a SDE:

$$\begin{aligned}
df = &\left(f_t + \mu S_t f_S - \sum_{i=1,2} \lambda_t^i \mathbb{E}(e^{J_i} - 1)S_t f_S\right. \\
&+ \left.\frac{1}{2}\sigma^2 S_t^2 f_{SS} + \sum_{i=1,2} \kappa_i(c_i - \lambda_t^i) f_{\lambda_i}\right) dt \\
&+ f_S \sigma\, dW_t + \left[f\left(t,\ S_t + S_t\left(e^{J_i} - 1\right),\ \lambda_t^1\right.\right. \\
&+ \left.\left.\delta_{1,1}J_1,\ \lambda_t^2 + \delta_{2,1}J_1\right) - f\right] dN_t^1 \\
&+ \left[f\left(t,\ S_t + S_t\left(e^{J_2} - 1\right),\ \lambda_t^1\right.\right. \\
&+ \left.\left.\delta_{1,2}J_2,\ \lambda_t^2 + \delta_{2,2}J_2\right) - f\right] dN_t^2.
\end{aligned} \tag{6}$$

It is then easy to show that if $f = \ln S_t$, the dynamics of the log stock index is provided by:

$$\begin{aligned}
d\ln S_t = &\left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma\, dW_t \\
&+ \sum_{i=1,2} \left(J_i dN_t^i - \mathbb{E}(e^{J_i} - 1)\lambda_t^i dt\right),
\end{aligned} \tag{7}$$

and that S_t is the product of two exponentials, one related to the Brownian motion and one to the jump processes of the log return:

$$\begin{aligned}
S_t = &S_0 \exp\left(\int_0^t \mu - \frac{1}{2}\sigma^2 ds + \int_0^t \sigma\, dW_s\right) \\
&\times \exp\left(\sum_{i=1,2} \int_0^t J_i dN_s^i - \sum_{i=1,2} \int_0^t \mathbb{E}(e^{J_i} - 1)\lambda_s^i ds\right).
\end{aligned} \tag{8}$$

On the other hand, we can show by direct integration of Eqs. (4) that λ_t^1 and λ_t^2 are given by:

$$\begin{aligned}
\lambda_t^1 = &c_1 + e^{-\kappa_1 t}\left(\lambda_0^1 - c_1\right) + \int_0^t e^{-\kappa_1(t-s)}\delta_{1,1}J_1 dN_s^1 \\
&+ \int_0^t e^{-\kappa_1(t-s)}\delta_{1,2}J_2 dN_s^2,
\end{aligned} \tag{9}$$
$$\begin{aligned}
\lambda_t^2 = &c_2 + e^{-\kappa_2 t}\left(\lambda_0^2 - c_2\right) + \int_0^t e^{-\kappa_2(t-s)}\delta_{2,1}J_1 dN_s^1 \\
&+ \int_0^t e^{-\kappa_2(t-s)}\delta_{2,2}J_2 dN_s^2.
\end{aligned}$$

These expressions allow us to calculate their expectation, detailed in the next proposition:

Proposition 2.1. *The expected values of λ_t^1 and λ_t^2 are given by*