Insurance: Mathematics and Economics 63 (2015) 108–120

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Mortality modelling with regime-switching for the valuation of a guaranteed annuity option

Huan Gao, Rogemar Mamon *, Xiaoming Liu, Anton Tenyakov

Department of Statistical and Actuarial Sciences, University of Western Ontario, London, Ontario, Canada

ARTICLE INFO

Article history:
Available online 17 April 2015

Keywords:
Gompertz model
Markov chain
Change of probability measure
Filtering
Exponential matrix

ABSTRACT

We consider three ways of putting forward a regime-switching approach in modelling the evolution of mortality rates for the purpose of pricing a guaranteed annuity option (GAO). This involves the extension of the Gompertz and non-mean reverting models as well as the adoption of a pure Markov model for the force of mortality. A continuous-time finite-state Markov chain is employed to describe the evolution of mortality model parameters which are then estimated using the filtered-based and least-squares methods. The adequacy of the regime-switching Gompertz model for the US mortality data is demonstrated via the goodness-of-fit metrics and likelihood-based selection criteria. A GAO is valued assuming the interest and mortality risk factors are switching regimes in accordance with the dynamics of two independent Markov chains. To obtain closed-form valuation formulae, we employ the change of measure technique with the pure endowment price as the numéraire. Numerical implementations are included to compare the results of the proposed approaches and those from the Monte Carlo simulations.

1. Introduction

Regime-switching (RS) models have gained popularity in modelling financial time series due to their ability to capture dynamic changes exhibited by the stochastic behaviour of economic and financial variables observed over time. Research works on RS models can be divided into two kinds: threshold models and Markov RS models. The difference between these two lies on the trigger of regime shifts, say, an observed variable for the former and a Markov chain for the latter, see Meyers (2009) for details. Goldfeld and Quandt (1973) proposed the switching regression model for housing markets data and this was viewed as one of the earliest applications of Markov RS models in economics. Markov-switching models in discrete time setting were extensively studied by Hamilton (1988, 1989) with special emphasis on economic and financial modelling. Since then, various interest rate models were developed that incorporate regime-switching characteristics either in the rate level itself or in the parameter dynamics. See Lewis (1991), Garcia and Perron (1996), Ang and Bekaert (2002), Mamon (2002), Elliott and Mamon (2002, 2003), Elliott and Siu (2009), amongst others. Moreover, RS models have been widely used in asset price modelling and pricing equity options, see for instance, Hardy (2001), Elliott et al. (2007) and Yuen and Yang (2010).

The increasing utility of regime-switching models in finance has influenced research on their applications in actuarial science. Milidonis et al. (2011) introduced a regime-switching approach to model mortality dynamics and highlighted favourable features of regime-switching models in mortality modelling. The distinguishing features of RS models include the capacity to identify structural changes and the flexibility to make parameter estimates change as time evolves depending on the dictates of the data. In their paper, they adopted a general RS model that switches between two geometric Brownian motions to model the annual US mortality index. In addition, they applied RS approach in modelling the time-varying mortality index under the Lee–Carter mortality framework.

Techniques of mortality modelling in practice have been traditionally deterministic before a variety of stochastic models have been developed to analyse the mortality improvements experienced in many countries, for instance the Lee–Carter model; see Cairns et al. (2009) for details. Since Gompertz (1825) first proposed the law of mortality asserting that the person's probability of dying increases at a constant exponential rate as age increases, many research works were developed based upon it, see for example, Trachtenberg (1924), Finch and Pike (1996) and Shklovskii (2005). It is well-accepted that the Gompertz's law holds between the ages of 30 and 90 over a wide time range of mortality data; see Spiegelman (1968) and Wetterstrand (1978). Tenenbein and

* Correspondence to: Department of Statistical and Actuarial Sciences, University of Western Ontario, 1151 Richmond Street, London, Ontario, Canada N6A 5B7.
E-mail address: rmamon@stats.uwo.ca (R. Mamon).

http://dx.doi.org/10.1016/j.insmatheco.2015.03.018
0167-6687/

Vanderhoof (1981) provided biophysical implications of the foundation of such a law. In Wetterstrand (1981), the ultimate mortality experience from life insurance for 1948–1977 was examined and changes of parameters in the law were described. On higher ages, say 90, the fluctuation of mortality is not easily dealt with by the Gompertz mortality law due to small sample sizes (cf. Bell and Miller, 2005). But, as mortality keeps on improving so that more people survive over 90 years old, thereby the small-sample size problem is mitigated, it is reasonable to expect that the exponential ageing law for humans is sustained at these ages. Some works on the modelling of mortality at higher ages include Bongaarts (2005) and Bebbington et al. (2014).

In this paper, the rationality of the Gompertz law is examined as our starting point using the US data from 1933–2009. The yearly mortality rates were found to follow an exponential increasing trend or the logarithm of the mortality rates has a linear form, which is in agreement with the Gompertz model; this result was determined using the regression method. Our analysis of the mortality patterns demonstrate dynamic changes in the parameters of the Gompertz model as the years go by. We, therefore, enrich the Gompertz model by putting forward a regime-switching model based on Gompertz law. We shall coin the term RS Gompertz model (RSGM) for the first new model proposed in this paper. Specifically, we employ a continuous-time Markov chain to modulate the parameters of the Gompertz model. The Markov chain captures the switching in the level of the rate governing the parameters' movement.

To provide data-based evidence of the capability of the proposed mortality model, we include an empirical study demonstrating the goodness-of-fit and likelihood-based measures supporting the superiority of the Markov-switching model compared to its one-regime counterpart. In particular, the hidden Markov model filtering approach described in Mamon et al. (2008) is employed and applied to US mortality data spanning a period of nearly eight decades to provide dynamic estimates of model parameters.

The primary goal in developing a mortality model with good adequacy is to support the pricing of insurance and annuity products. Many product innovations in recent times were introduced, and their pricing and reserving present new challenges. It is now common for products to have investment guarantees in them and therefore, they involve embedded options (cf. Bolton et al., 2004, and Boyle and Hardy, 1997). The stochastic modelling of two key factors, namely, the interest and mortality rates (see Ballotta and Haberman, 2006, and Milevsky and Promislow, 2001) is the most important consideration in valuation. Although there have been substantial research achievements in insurance pricing under a stochastic environment as elaborated in Hardy (2003) and Dahl (2004), the pricing of products with long-dated horizons, under a consistent framework in which both interest and mortality rates are regime-switching, needs further development. This work constructs a new framework whereby the interest rate process follows a pure Markov RS model whilst the mortality rates are deemed to follow a regime-switching mortality model. The pricing, hedging and reserving problem for guaranteed annuity options (GAOs) were studied in Wilkie et al. (2003) and Liu et al. (2013, 2014), amongst others. In this paper, we further propose two alternative RS mortality models motivated by the limitation of the RSGM, which does not provide analytical solution to the survival probability. We then consider the GAO pricing along with a numerical implementation and investigation of price sensitivity to parameters of a combined regime-switching models under the two alternatives. Aiming for an analytic pricing solution of GAO, we develop a Gompertz model with RS regression parameters as well as extend the methodology in Elliott and Mamon (2003). We then derive the endowment price under the assumption that the two factors are driven by independent Markov chains under each alternative modelling set-up. Consequently, we use the so-called endowment-risk-adjusted measure, which was first introduced in Liu et al. (2013) and show the significant benefits of the change of measure technique over Monte-Carlo-based implementation. In addition, we show the flexibility of RS models in accommodating a range of underlying states in our model framework by changing the states of each random factor.

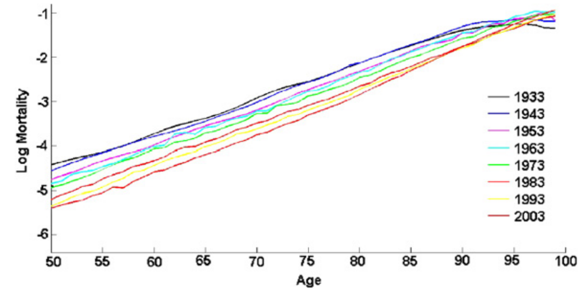

Fig. 1. US log-mortality rate in different years for the period 1933–2009 consistent with model in Eq. (1).

The essence of this paper is to offer three ways of formulating a regime-switching mortality framework, where the first framework is well-supported by the data, and the other two are less sophisticated than the first one but they are designed to produce GAO analytic pricing solutions. To attain our objectives, this paper is organised as follows. Section 2 presents the proposed three regime-switching mortality models. An analysis of the mortality trend using the US data for the period 1933–2009 validates the utility of the RSGM. Two alternative RS models are provided along with their analytical solutions to the survival index. The pure Markov interest rate model is described including the derivation of bond price in Section 3. In Section 4, we formulate the RS framework to value a GAO by integrating a Markov interest rate model and the alternative mortality models put forward in this paper. Interest and mortality processes are governed by two independent Markov chains for tractability. As will be illustrated, the measure-change method facilitates efficiently the pricing implementation. Numerical results are given in Section 5. Some remarks and an indication of future works conclude this paper in Section 6.

2. Mortality models

2.1. Model 1 (M1): Gompertz model with BM- and Markov-switching parameters

2.1.1. Mortality analysis

The stochasticity of mortality rate is documented in Tuljapurkar and Boe (1998) and Pitacco (2004). A proposed model aimed to capture salient features of a mortality process must be supported by empirical evidence. So, before embarking on the full development of a mortality model, we first examine the US mortality data from 1933 to 2009 available and downloadable from the Human Mortality Database (URL: www.mortality.org). Since we are aiming at pricing pension products, we focus on older ages, in particular, from 50 to 100.

We display the log mortality rates through age for certain specific years in Fig. 1, and the graph shows an approximately linear trend each year. This, consistent with the traditional Gompertz law, is based on the model that for a life aged x at time t, the force of mortality model is given by

$$\log \mu(x, t) = a(t) + b(t)x. \quad (1)$$