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A common age effect model for the mortality of multiple populations

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ABSTRACT

We introduce a model for the mortality rates of multiple populations. To build the proposed model we investigate to what extent a common age effect can be found among the mortality experiences of several countries and use a common principal component analysis to estimate a common age effect in an age–period model for multiple populations. The fit of the proposed model is then compared to age–period models fitted to each country individually, and to the fit of the model proposed by Li and Lee (2005).

Although we do not consider stochastic mortality projections in this paper, we argue that the proposed common age effect model can be extended to a stochastic mortality model for multiple populations, which allows to generate mortality scenarios simultaneously for all considered populations. This is particularly relevant when mortality derivatives are used to hedge the longevity risk in an annuity portfolio as this often means that the underlying population for the derivatives is not the same as the population in the annuity portfolio.

1. Introduction

A number of stochastic models for mortality rates were developed in recent years. Among those the Lee–Carter (LC) model introduced by Lee and Carter (1992) remains a very popular and widely used model. This model breaks down the mortality experiences at different ages and calendar years into age and period effects. The period effect for a given population can then be viewed as a mortality index for all ages. When a LC model is fitted to a number of populations individually, an individual age effect is obtained for each population. This makes it more difficult to compare the period effects observed in different populations as they are fitted to different age effects.

In this paper we consider an extension of the LC model to multiple populations where the age effect is common to all populations. We will call this model a common age effect (CAE) model. In particular, we study the differences in the goodness of fit between individual models and CAE models. The main question we wish to answer is: how important are individual age effects for the goodness of fit of individual LC models compared to the impact of an additional age–period effect in a CAE model?

This study is motivated by the observation that obtained age effects are very similar when they are estimated in different countries of similar socio-economic structure. This suggests that the number of parameters, in particular, age effects, can be reduced when the mortality experiences of several countries or populations are modelled simultaneously. In addition, a CAE model allows for more direct comparison of period effects, since these period effects in different populations are scaled with the same age parameters.

The proposed model can be applied directly to mortality data from different countries or populations, or, alternatively, can be applied to the residuals of other multiple population models, for example, the multiple population model introduced by Kleinow and Cairns (2013) where smoking prevalence is used to explain differences in the mortality experiences in different countries.

In addition to the introduction of the CAE model we also show how to use an estimation method called common principal component analysis to identify common age effects. The proposed model can be fitted using other estimation methods like Maximum Likelihood Estimation. However, using common PCA has some advantages, which we discuss in Section 3.

In our empirical study we will apply the model to the mortality rates observed for males aged 18–87 in the following ten countries: Austria, Australia, Canada, Switzerland, Denmark, France, Great Britain, New Zealand, Sweden and the United States. We choose those ten countries since they are all well developed countries with similar socio-economic characteristics. Therefore, we expect that a mortality model with common factors will allow us to jointly model mortality rates in those countries. The empirical results are based on observed mortality rates for the calendar years 1948–2007. We will split the ages into two groups of 35 years each,

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that is, we separately consider males aged 18–52 and 53–87. This is necessary since we require the number of calendar years to exceed the number of ages. All observed mortality rates are obtained from the Human Mortality Database.

In Section 2 we will review the LC model including a straight forward extension to p age–period effects. This also includes a brief review of the estimation of parameters using principal component analysis (PCA) rather than maximum likelihood methods. We concentrate here on the PCA as we wish to use a modification of this method, called common principal component analysis (cPCA) in Section 3 to obtain estimates of the common age effects. In the following Section 4 we will then compare the estimated age and period effects resulting from the individual models and the CAE model. In the same section we will also compare the goodness of fit of the two models.

2. Individual model

We consider the mortality rates in k populations. For each population $i = 1, \ldots, k$ we observe the realised log mortality rates $\tilde{m}_i(x, t)$ at age $x \in \{x_1, \ldots, x_n\}$ in year $t = 1, \ldots, T$, that is,

$$\tilde{m}_i(x, t) = \log \frac{D_i(x, t)}{E_i(x, t)}$$

where $D_i(x, t)$ is the observed number of deaths in country i at age x during year t and $E_i(x, t)$ is the corresponding exposure to risk. These rates are observed for n different ages and a total of T years. We assume that $T > n$, and the ages $x_1, \ldots, x_n$ and the years $1, \ldots, T$ are the same for all populations.

In the following we will consider centralised log mortality rates. Therefore, we first calculate the average log mortality rate, $\bar{m}_i(x)$, for a life aged x in population i, that is,

$$\bar{m}_i(x) = \frac{1}{T} \sum_{t=1}^{T} \tilde{m}_i(x, t)$$

and define the centralised log mortality rates

$$m_i(x, t) = \tilde{m}_i(x, t) - \bar{m}_i(x).$$

We denote by m_i the matrix of the observed centralised log mortality rates, that is,

$$m_i = \begin{pmatrix} m_i(x_1, 1) & \cdots & m_i(x_1, T) \\ \vdots & & \vdots \\ m_i(x_n, 1) & \cdots & m_i(x_n, T) \end{pmatrix}.$$

The individual model of order p for the centralised mortality rates m_i in each country i is an extension of the Lee–Carter model to p age and period effects, that is,

$$m_i(x, t) = \beta_i^{(1)}(x)\kappa_i^{(1)}(t) + \cdots + \beta_i^{(p)}(x)\kappa_i^{(p)}(t) + \varepsilon_i(x, t)$$

which can be written in matrix form as:

$$m_i = {}_p\beta_i {}_p\kappa_i + \varepsilon_i \tag{1}$$

with

$${}_p\beta_i = \begin{pmatrix} \beta_i^{(1)}(x_1) & \cdots & \beta_i^{(p)}(x_1) \\ \vdots & & \vdots \\ \beta_i^{(1)}(x_n) & \cdots & \beta_i^{(p)}(x_n) \end{pmatrix} \quad \text{and}$$
$${}_p\kappa_i = \begin{pmatrix} \kappa_i^{(1)}(1) & \cdots & \kappa_i^{(1)}(T) \\ \vdots & & \vdots \\ \kappa_i^{(p)}(1) & \cdots & \kappa_i^{(p)}(T) \end{pmatrix}. \tag{2}$$

The residuals $\varepsilon_i = \left(\varepsilon_i(x, t)\right)$ form a $n \times T$ matrix, and we assume that $\mathrm{E}[\varepsilon_i(x, t)] = 0$ for all populations i. To avoid identifiability problems we also assume that $\|\beta_i^{(j)}\| = 1$ for all i and j, where $\|.\|$ denotes the Euclidean norm, that is, $\|x\| = x^\top x$ for any vector x. The maximum number of age effects is $p = n$ since there are only n ages. To simplify notation we define

$$\beta_i = {}_n\beta_i.$$

The individual model can be fitted in different ways. In the actuarial literature, methods based on Maximum Likelihood Estimation (assuming a particular distribution for the number of deaths) are widely used. Alternatively, methods based on generalised linear models could also be applied. Since those methods are based on models for the number of deaths rather than models for the mortality rates, the obtained estimates for the age and period effects are strongly dependent on those ages and periods in which large numbers of deaths have been observed, and less dependent on ages and periods in which relatively few deaths have been observed. This is often seen as an advantage. However, we wish to extend the individual model to a model for multiple populations that are of different sizes. We therefore prefer a method that attaches the same weight to all observed mortality rates.

It is well known that estimates for $\beta_i^{(j)}$ (column j in matrix β_i) for any individual population i can also be obtained by a principal component analysis using a singular value decomposition of the matrix m_i, that is,

$$m_i = \beta_i L_i U_i^\top$$

where β_i is a $n \times n$ orthogonal matrix, that is, $\beta_i^\top \beta_i$ is the n-dimensional identity matrix, L_i is a $n \times n$ diagonal matrix, and U_i is a $T \times n$ matrix with mutually orthonormal columns, that is, $U_i^\top U_i$ is the n-dimensional identity matrix. We assume that all matrices m_i have full rank, which is then equal to n since we assumed that $n < T$. Note that the singular value decomposition above can also be stated in terms of a $n \times T$ diagonal matrix L_i, and a $T \times T$ orthogonal matrix U_i. Such a decomposition would be equivalent to the one used here.

Also note that the estimated matrix of age effects is now an orthogonal matrix, meaning that the identifiability constraint $\|\beta_i^{(j)}\| = 1$ is fulfilled, and, in addition, $\beta_i^{(j)\top} \beta_i^{(l)} = 0$, which is, in general, not the case if age effects are estimated using maximum likelihood methods.

Equivalently, estimates for β_i can also be obtained from computing the eigenvectors of $m_i m_i^\top$:

$$Q_i = m_i m_i^\top = \beta_i \kappa_i \kappa_i^\top \beta_i^\top = \beta_i \Lambda_i \beta_i^\top$$

since

$$\begin{aligned} m_i m_i^\top &= \beta_i L_i U_i^\top U_i L_i^\top \beta_i^\top = \beta_i L_i L_i^\top \beta_i^\top \\ &= \beta_i \Lambda_i \beta_i^\top \quad \text{with } \Lambda_i = L_i L_i^\top. \end{aligned}$$

The eigenvalues of $m_i m_i^\top$ are on the diagonal of the matrix Λ_i, and the first estimated age effect $\hat{\beta}_i^{(1)}$ is then the eigenvector corresponding to the largest eigenvalue of $m_i m_i^\top$. For an individual model of order $p \leq n$ we only use the p estimated eigenvectors corresponding to the p largest eigenvalues, that is, the estimated matrix ${}_p\hat{\beta}_i$ contains the first p columns of $\hat{\beta}_i$.

The estimated first age effects $\hat{\beta}_i^{(1)}$ for the ten countries mentioned in the introduction are shown in Fig. 1 in grey. In can be seen in this figure that the age effects for ages 53–87 are indeed rather similar for different countries and might therefore be replaced by an age effect that is the same for all countries. For younger ages this is less obvious. We will now turn to a model and a corresponding estimation procedure for such a common age effect. The black line in Fig. 1 already shows the estimated first common age effect for these countries based on the CAE model that we will introduce in the following section.