Insurance: Mathematics and Economics 63 (2015) 169–190

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Prospective mortality tables: Taking heterogeneity into account[☆]

Julien Tomas [*,1], Frédéric Planchet

ISFA, Laboratoire SAF, France[2]

ARTICLE INFO

Article history:
Available online 22 June 2015

Keywords:
Heterogeneity
Prospective mortality table
Local likelihood
Singular values decomposition
Cox model
Relational models
Life insurance

Mots clés:
Hétérogénéité
Table de mortalité prospective
Vraisemblance locale
Décomposition en valeurs singulières
Modèle de Cox
Modèles relationnels
Assurance vie

ABSTRACT

The present article illustrates an approach to construct prospective mortality tables for which the data available are composed by heterogeneous groups observed during different periods. Without explicit consideration of heterogeneity, it is necessary to reduce the period of observation at the intersection of the different populations observation periods. This reduction of the available history can arm the determination of the mortality trend and its extrapolation. We propose a model taking explicitly into account the heterogeneity, so as to preserve the entire history available for all populations. We use local kernel-weighted log-likelihood techniques to graduate the observed mortality. The extrapolation of the smoothed surface is performed by identifying the mortality components and their importance over time using singular values decomposition. Then time series methods are used to extrapolate the time-varying coefficients. We investigate the divergences in the mortality surfaces generated by a number of previously proposed models on three levels. These concern the proximity between the observations and the model, the regularity of the fit as well as the plausibility and consistency of the mortality trends.

RÉSUMÉ

Cet article illustre une approche concernant la construction d'une table de mortalité prospective pour laquelle les données disponibles sont constituées de groupes a priori hétérogènes et observés sur des périodes diffrentes. Sans prise en compte explicite de l'hétérogénéité, il est nécessaire de réduire la période d'observation à l'intersection des périodes d'observation des diffèrentes populations. Cette réduction de l'historique disponible s'avère pénalisant pour la détermination des tendances d'évolution de la mortalité et ainsi son extrapolation. Nous proposons un modèle intégrant explicitement la prise en compte de l'hétérogénéité, à partir du modèle de Cox, pour permettre de conserver l'ensemble de historique disponible pour toutes les populations. Nous utilisons des méthodes non-paramétriques de vraisemblance locale pour graduer la mortalité observée. L'extrapolation de la surface ajustée est obtenue en identifiant dans en premier temps les composantes de la mortalité et leur importance dans le temps par une décomposition en valeurs singulières. Des méthodes de séries temporelles sont employées pour extrapoler les paramètres variant dans le temps. Nous analysons les divergences observées entre les surfaces de mortalité générées sur trois niveaux. Ceux-ci concernent la proximité entre les observations et le modèle, la régularité de l'ajustement ainsi que la plausibilité et la cohérence des tendances d'évolution de la mortalité.

1. Introduction

In this article, we present an approach to construct prospective mortality tables for which the data available are composed

☆ This work has been supported by the BNP Paribas Cardif Chair "Management de la modélisation" and the French Institut des Actuaires. The views expressed in this document are the authors own and do not necessarily reflect those endorsed by BNP Paribas Cardif and Institut des Actuaires.

* Corresponding author.
E-mail addresses: julien.tomas@univ-lyon1.fr (J. Tomas), frederic.planchet@univ-lyon1.fr (F. Planchet).

1 All computer programs are available on request.

2 Institut de Science Financière et d'Assurances, Université Claude Bernard Lyon 1, 50 Avenue Tony Garnier, 69366 Lyon, France

http://dx.doi.org/10.1016/j.insmatheco.2015.03.020
0167-6687/

by heterogeneous groups observed during different periods. The approach is motivated by having the largest available history to determine the mortality trends.

It has been observed that the human mortality has globally declined over the 20th century. Life expectancy is greater than ever before and continues to improve rapidly, see Pitacco et al. (2009, Chapter 3). These improvements affect the pricing and reserving in life insurance and constitute a challenge for actuaries and demographers in modeling the longevity.

In a pension plan, the longevity risk is transferred from the policyholder to the insurer. The latter has to evaluate his liability with appropriate mortality tables. It is in this context that since 1993 the French regulatory tables for annuities have been dynamic taking in account the increase of the life expectancy.

Dynamic (or prospective) mortality tables allow to determine the remaining lifetime for a group, not according to the conditions of the moment, but given the future developments of living conditions.

However, applying exogenous tables to the group considered may result in under-provisioning the annuities, when the mortality of the group is lower than of the reference population.

With the international regulations *Solvency II* and *IFRS* insurers are required to evaluate their liabilities from realistic assumptions leading to an evaluation of the *best estimate*. In consequence, for pensions regimes and more generally due to the longevity risk, insurers have to build specific mortality tables, taking into account the expected evolution of the mortality of their insured population, see Planchet and Kamega (2013). It is in this context that we apply our approach to the construction of a reference mortality table from portfolios of several insurance companies. This reference could be used to adjust the mortality specifically to each insured portfolio and construct entity specific dynamic mortality tables.

We are in the situation where the data available are composed by heterogeneous groups observed during different periods. Without explicit consideration of heterogeneity, it is necessary to reduce the period of observation at the intersection of the different populations observation periods. This reduction of the available history can arm the determination of the mortality trend and its extrapolation. We propose a model taking explicitly into account the heterogeneity so as to preserve the entire history available for all populations. The innovative aspect lies in the articulation of a Cox model in a preliminary step and methods to graduate and extrapolate the mortality to construct a mortality table summarizing the mortality experience of all populations. We use local kernel-weighted log-likelihood techniques to graduate the observed mortality in a second step. The extrapolation of the smoothed surface is then performed by identifying the mortality components and their importance over time using singular values decomposition. The number of parameters is determined according to their explicative power. Then time series methods are used to extrapolate the time-varying coefficients.

This article is organized as follows. Section 2 has still an introductory purpose. It specifies the notation and assumptions used in the following. Section 3 describes our approach to take explicitly into account the heterogeneity in constructing prospective mortality tables. Section 4 presents an application concerning the construction of a reference table from portfolios of various French insurance companies. Finally, some remarks in Section 5 conclude the paper.

2. Notation, assumption

2.1. Notation

We analyze the mortality as a function of both the attained age x and the calendar year t. The force of mortality at attained age x for the calendar year t is denoted by $\varphi_x(t)$. We denote $D_{x,t}$ the number of deaths recorded at attained age x during calendar year t from an exposure-to-risk $E_{x,t}$ that measures the time during individuals are exposed to the risk of dying. It is the total time lived by these individuals during the period of observation. We suppose that we have data line by line originating from a portfolio. To each of the observations i, we associate the dummy variable δ_i indicating if the individual i dies or not,

$$\delta_i = \begin{cases} 1 & \text{if individual } i \text{ dies,} \\ 0 & \text{otherwise,} \end{cases}$$

for $i = 1, \ldots, L_{x,t}$. We define the time lived by individual i before $(x+1)th$ birthday by τ_i. We assume that we have at our disposal i.i.d. observations (δ_i, τ_i) for each of the $L_{x,t}$ individuals. Then,

$$\sum_{i=1}^{L_{x,t}} \tau_i = E_{x,t} \quad \text{and} \quad \sum_{i=1}^{L_{x,t}} \delta_i = D_{x,t}.$$

2.2. Piecewise constant forces of mortality

We assume that the age-specific forces of mortality are constant within bands of time, but allowed to vary from one band to the next, $\varphi_{x+\tau}(t+\xi) = \varphi_x(t)$ for $0 \le \tau < 1$ and $0 \le \xi < 1$.

We denote by $p_x(t)$ the probability that an individual aged x in calendar year t reaches age $x+1$, and by $q_x(t) = 1 - p_x(t)$ the corresponding probability of death. The expected remaining lifetime of an individual reaching age x during calendar year t is denoted by $e_x(t)$.

Under the assumption of piecewise constant forces of mortality, we have for integer age x and calendar year t,

$$p_x(t) = \exp\big(-\varphi_x(t)\big) \quad \text{and} \quad \varphi_x(t) = -\log\big(p_x(t)\big).$$

3. The approach

Our approach can be summarized as follows:

i. From a proportional hazard model, we describe how the risk of the populations changes over time. The resulting coefficients are used in the following step to weight the exposure to risk of each population.
ii. We smooth the surface using non-parametric local kernel-weighted log-likelihood to estimate $\varphi_x(t)$ for $x \in [x_1, x_n]$ and $t = 1, \ldots, m$.
iii. We decompose the smoothed surfaces via a basis function expansion using the following model:

$$\log \widehat{\varphi}_x(t) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k}\, \phi_k(x) + \varepsilon_t(x)$$

$$\text{with } \varepsilon_t(x) \sim \text{Normal}\big(0, \upsilon(x)\big), \tag{1}$$

where $\mu(x)$ is the mean of $\log \widehat{\varphi}_x(t)$ across years and $\{\phi_k(x)\}$ is a set of orthonormal basis functions.
iv. ARIMA models are fitted to each of the coefficients $\{\beta_{t,k}\}$, $k = 1, \ldots, K$.
v. We extrapolate the coefficients $\{\beta_{t,k}\}$, $k = 1, \ldots, K$, for $t = m+1, \ldots, m+h$ using the fitted time series models.
vi. Finally, we use the resulting forecast coefficients with (1) to obtain forecasts of $\varphi_x(t)$, $t = m+1, \ldots, m+h$.

3.1. Taking into account heterogeneity

In step i., we propose to describe from a proportional hazard model how the risk of the populations changes over time. Assuming the proportional hazards assumption holds then it is possible