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What attitudes to risk underlie distortion risk measure choices?

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ABSTRACT

Understanding the attitude to risk implicit within a risk measure sheds some light on the way in which decision makers perceive losses. In this paper, a two-stage strategy is developed to characterize the underlying risk attitude involved in a risk evaluation, when executed by the family of distortion risk measures. First, we show that aggregation indicators defined for Choquet integrals provide information about the implicit global risk attitude of the agent. Second, an analysis of the distortion function offers a local description of the agent's stance on risk in relation to the occurrence of accumulated losses. Here, the concepts of *absolute risk attitude* and *local risk attitude* arise naturally. An example is provided to illustrate the usefulness of this strategy for characterizing risk attitudes in an insurance company.

1. Introduction

1.1. Motivation

Tools designed to provide adequate risk measurements are needed by both decision-making agents and regulatory agents, who require information about potential losses within a probabilistic framework. As such, the choice of a risk measure plays a central role in decision-making in many areas including health, safety, environmental, adversarial and catastrophic risks (Cox, 2013; MacKenzie, 2014). Many different risk measures are available to practitioners, but the selection of the most suitable risk measure for use in a given context is generally controversial. A key element in characterizing a risk measure is the underlying risk attitude that is assumed when this measure is used for risk assessment. Clearly, therefore, in selecting the best measure, the practitioner is concerned with how a particular measure matches up with the alternatives. However, this simple question is only satisfied with a complex answer.

Consider the Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR), probably the most common measures used in assessing risk. Suppose α is the confidence level, which reflects the degree of tolerance to undesirable events. The $\mathrm{VaR}_\alpha(X)$ is the α-quantile of loss X, while the $\mathrm{TVaR}_\alpha(X)$ averages quantiles ranging from the α-quantile to the maximum (the 100%-quantile) of X. Based on these definitions, it seems obvious that these two quantile-based risk measures can be directly compared in terms of their respective conceptions of risk using their associated confidence levels. For instance, the Value-at-Risk measure provides for a concept of risk associated with a barrier, beyond which the decision maker assumes catastrophe lies (Alexander and Sarabia, 2012). A Value-at-Risk measure at a 95% confidence level presents a lower resistance to undesirable events than a VaR measure at a 99% level. This also holds for $\mathrm{TVaR}_\alpha(X)$. Comparisons of VaR and TVaR measures can likewise be readily undertaken when their respective confidence levels are fixed and equal. Given an α-confidence level, the $\mathrm{TVaR}_\alpha(X)$ is always greater or equal than the $\mathrm{VaR}_\alpha(X)$. However, a direct comparison cannot be made if the VaR and the TVaR risk measures have different confidence levels. For example, imagine a decision maker wishes to compare the implicit risk attitude of the $\mathrm{TVaR}_{95\%}(X)$ and the $\mathrm{VaR}_{99\%}(X)$. In this instance, it is not immediately obvious which of these two risk measures offers the greatest risk tolerance. Furthermore, if the decision maker wants to know the risk attitude of a measure other than that of these two quantile-based measures, comparisons are even less intuitive.

Here, we focus on the family of distortion risk measures, where the VaR and TVaR can be understood as two particular cases. A battery of instruments is developed to facilitate the comparison of the risk attitude of distortion risk measures from both global and local perspectives. The results afford new elements

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for determining the suitability of a particular distortion risk measure in comparison with other available options. They also allow an agent to determine which risk measure provides the most risk tolerant behavior.

1.2. Attitude towards risk

This article seeks to contribute to the study of attitudes towards risk in the assessment of risk. The study analyses the risk perception that is implicit when an agent applies a particular distortion risk measure. The characterization of the implicit attitude towards risk in a given distortion risk measure is carried out by means of the computation of aggregation indicators and an analysis of the distortion function. The combination of these two instruments provides a precise portrait of the underlying risk position of a decision-maker when selecting a particular risk measure for risk assessment.

Distortion risk measures can be represented mathematically as a class of Choquet integrals (Wang, 1995; Belles-Sampera et al., 2013; Grigorova, 2014). One way in which to describe the characteristics of Choquet integrals is to use a set of aggregation indicators, which provide information about features of the underlaying aggregation operator (Marichal, 2004; Beliakov et al., 2007; Belles-Sampera et al., 2014c; Yager, 2000; Kojadinovic et al., 2005; Grabisch et al., 2009). Here, we investigate the quantitative information related to the overall risk attitude associated with the risk measure as provided by the aggregation indicators. It is our contention that these indicators are useful for characterizing the global perception of risk implicit in the risk measure choice.

It is reasonable to suppose that decision-makers do not worry about all random event losses in the same way. Decision makers frequently treat different random events distinctly (note that some of these events can represent benefits or affordable losses). Therefore, the global vision of risk embedded in a risk measure has to be completed with local information. In this paper, we define a quotient function, based on the distortion function associated with the risk measure, in order to characterize the local vision of risk. The quotient function is graphically analyzed to investigate the risk attitude of the agent at any point in the survival distribution function when using a certain risk measure. The graphical evaluation of the risk-appetite pattern of a manager in the range of feasible values is the basis of the definition of two concepts: *absolute risk attitude* and *local risk attitude*.

The attitudes to risk implicit within three particular distortion risk measures are studied. Our attention is focused on the characterization of the attitudes toward risk of the VaR, TVaR and a class of four-parameter distortion risk measures that are called GlueVaR (Belles-Sampera et al., 2014a). The high flexibility of the GlueVaR distortion measures allows different specific attitudes to be reflected. We examine the additional risk information provided by these risk measures and their usefulness for decision makers.

An illustrative example of the risk attitude characterization implicit in a distortion risk measure is included in this article. The European insurance regulatory framework serves as an excellent example of the choice of a compulsory risk measure, i.e. $\mathrm{VaR}_{99.5\%}$. However, insurers implement other choices in their internal tools. We show that, given a particular insurer's dataset, distortion risk measures other than that of the Value-at-Risk can provide the same risk estimates. However, if the insurer does chose a different risk measure, this provides complementary tools for evaluating risk that can be used to understand its position in the European insurance or financial market, or even to benchmark it in relation to the mandatory risk assessment standard.

The article is structured as follows. Section 2 is devoted to a brief presentation of distortion risk measures and indicators for Choquet integrals. Section 3 examines and discusses the tools to analyze implicit risk attitudes in distortion risk measures. Section 4 describes an application and its results, and outlines the strategy and the methodology used to calibrate risk measure parameters. The programming of the data analysis was carried out using the open source R statistical programming language and software. Finally, Section 5 concludes.

2. Distortion risk measures and aggregation indicators

2.1. Choquet integral and distortion risk measures

We define the Choquet integral in line with Denneberg (1994). The (asymmetric) Choquet integral with respect to a set function μ of a μ-measurable function $X : \Omega \to \overline{\mathbb{R}}$ is denoted as $\mathcal{C}_\mu(X)$ and is equal to

$$\mathcal{C}_\mu(X) = \int X d\mu = \int_{-\infty}^{0} \left[S_{\mu,X}(x) - \mu(\Omega)\right] dx + \int_{0}^{+\infty} S_{\mu,X}(x) dx, \qquad (1)$$

if $\mu(\Omega) < \infty$, where $S_{\mu,X}(x) = \mu(\{X > x\})$ denotes the *survival function* of X with respect to μ. Note that Ω denotes a set, which in many applications is the sample space of a probability space. A set function μ in this context is a function defined from 2^Ω (the set of all subsets of Ω) to $\overline{\mathbb{R}}$. A μ-measurable function X is, widely speaking, a function defined on Ω so that expressions like $\mu(\{X > x\})$ or $\mu(\{X \le x\})$ make sense. If μ is defined so that $0 \le \mu(\Omega) < \infty$ and it also satisfies that $\mu(\emptyset) = 0$ and that if $A \subseteq B$ then $\mu(A) \le \mu(B)$, for any $A, B \in 2^N$ (monotonicity), then μ is often called a *capacity*.

The Choquet integral may be used in the definition of distortion risk measures as introduced by Wang (1995, 1996). A distortion function is a non-decreasing and injective function g from [0, 1] to [0, 1] such that $g(0) = 0$ and $g(1) = 1$. Consider a probability space and the set of all random variables defined on this space. Given one of these random variables X, the value $\rho_g(X)$ that a distortion risk measure returns when applied to X may be understood as the value of the asymmetric Choquet integral of X with respect to a capacity, which is built by distorting the survival probability of X with the distortion function g, i.e. $\rho_g(X) = \int X d(g \circ P)$.

Therefore, the distortion risk measure can be defined as $\rho_g(X) = \mathcal{C}_\mu(X)$ with $\mu = g \circ P$, as shown in (1). Note that the distortion function g distorts the survival probability of X. The mathematical expectation of X can be understood as a particular case of a distortion risk measure such that $\mathbb{E}(X) = \mathcal{C}_{id \circ P}(X)$, where the distortion function is the identity function *id*. Indeed, the value of a distortion risk measure $\rho_g(X)$ may be interpreted as the expectation of X given that the survival probability of X has been previously distorted by the function g.

The most frequently used distortion risk measures are the quantile-based risk measures Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR). Let us consider the VaR_α and TVaR_α risk measures that are defined for a random variable X as $\mathrm{VaR}_\alpha(X) = \inf\{x \mid F_X(x) \ge \alpha\} = F_X^{-1}(\alpha)$ and $\mathrm{TVaR}_\alpha(X) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_\lambda(X)\, d\lambda$, respectively, where $0 \le \alpha \le 1$ is a confidence level. The relationship between the VaR and TVaR risk measures and discrete Choquet integrals has been described in literature (Belles-Sampera et al., 2013). Both $\mathrm{VaR}_\alpha(X)$ and $\mathrm{TVaR}_\alpha(X)$ may be understood as Choquet integrals with respect to capacities $\nu = \psi_\alpha \circ P$ and $\tau = \gamma_\alpha \circ P$, respectively, where P is the probability function of X, $\psi_\alpha(u) = \mathbb{1}_{[1-\alpha,1]}(u)$ and $\gamma_\alpha(u) = \frac{u}{1-\alpha} \cdot \mathbb{1}_{[0,1-\alpha)}(u) + \mathbb{1}_{[1-\alpha,1]}(u)$ are the distortion functions associated with these risk measures, where $\mathbb{1}_I(u)$ denotes an indicator function which equals 1 when u is in the interval I and 0 otherwise.