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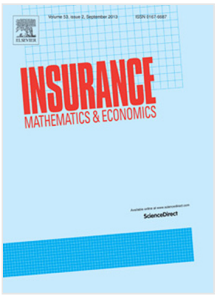

Nonlinear reserving in life insurance: Aggregation and mean-field approximation

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Nonlinear reserving in life insurance: aggregation and mean-field approximation

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Abstract

We suggest a unified approach to claims reserving for life insurance policies with reserve-dependent payments driven by multi-state Markov chains. The associated prospective reserve is formulated as a recursive utility function using the framework of backward stochastic differential equations (BSDE). We show that the prospective reserve satisfies a nonlinear Thiele equation for Markovian BSDEs when the driver is a deterministic function of the reserve and the underlying Markov chain. Aggregation of prospective reserves for large and homogeneous insurance portfolios is considered through mean-field approximations. We show that the corresponding prospective reserve satisfies a BSDE of mean-field type and derive the associated nonlinear Thiele equation.

Keywords: Backward stochastic differential equation, Life insurance, Multistate models, Markov process, Mean-field, Surrender value, Thiele's equation.

1 Introduction

Policies with reserve-dependent payments arise naturally in life insurance and pension. A typical example is a pension where the benefits less premiums include a cost of capital fee that is proportional to the reserve. Other examples include life insurance policies with withdrawal options, where the policyholder pays a lump sum fee proportional to the reserve in the case of withdrawal. In both cases, the payments from the policy are linear functions of the reserve. The problem of determining the corresponding retrospective and prospective reserves has been considered by several authors. Norberg [12] lists several examples, including a widow's pension policy in the presence of administration expenses that depend partly of the reserve, and derives explicit solutions to the corresponding Thiele equations. Christiansen *et al.* [4] consider reserve-dependent payments and derive explicit expressions for the reserve for the special case when the payment functions are linear in the reserve.

Claims reserving for policies where the payments depend in a nonlinear way of the reserve has not received the same attention. Examples of such policies include guaranteed benefits, where the benefits depend on the maximum of the accumulated reserve and a guaranteed amount. Christiansen *et al.* [4] briefly consider this example, and note that, in this case, the corresponding Thiele equation has to be solved using numerical methods.

Another important consideration for insurance companies is that of diversification and aggregation of insurance portfolios. Consider, for example, policies where, at the death of one policyholder, (a proportion of) the accumulated reserve of that policy is distributed back to the collective. The payments will then depend on the size of the reserve of each policy relative to that of the 'average' policy (aka model point), or the mean size of the reserves of all policies, in the case of a very large and homogeneous portfolio. This example motivates for the concept of aggregation and mean-field approximation for life insurance contracts

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