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Pricing and hedging basket options with exact moment matching

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ABSTRACT

Theoretical models applied to option pricing should take into account the empirical characteristics of financial time series. In this paper, we show how to price basket options when the underlying asset prices follow a displaced log-normal process with jumps, capable of accommodating negative skewness and excess kurtosis. Our technique involves Hermite polynomial expansion that can match exactly the first m moments of the model-implied basket return. This method is shown to provide superior results for basket options not only with respect to pricing but also for hedging.

1. Introduction

Basket options are contingent claims on a group of assets such as equities, commodities, currencies and even other vanilla derivatives. They are a subclass of exotic options and commonly traded over-the-counter in order to hedge away exposure to correlation or contagion risk. Additionally, they are also employed by hedge-funds for investment purposes, to combine diversification with leverage.

Baskets consist of several assets and, consequently, any modelling ought to be multidimensional. Many pricing models that seem to work well for single assets cannot be easily extended to a multidimensional set-up, mainly due to computational difficulties. The major problem is that in many cases the probability density function of the basket values at expiration is not known. Hence, practitioners usually resort to classic multidimensional geometric Brownian motion type models to keep the modelling framework as simple as possible. However, by doing so, the computational problems are not completely solved because the probability density function of the sum of log-normal variables is not known and additionally the empirical characteristics of the assets in the basket are simply overlooked. In particular, the negative skewness and excess kurtosis, which are well known to characterize equities, cannot be captured properly by these simple models because they can produce a limited range of values for these statistics.

Ideally, one would like the best of both worlds, realistic modelling and precise calculations. In this paper, we present a general computational solution to the problem of multidimensional models lacking closed-form formulae or requiring burdensome numerical procedures. The purpose of this paper is to provide a robust and precise methodology for pricing and hedging basket options when the price of each of the assets in the basket follows a model able to accommodate the empirical characteristics. One such model is the displaced jump–diffusion which will be used as test subject to show the superiority of the presented methodology. This model is very useful for the dynamics of one asset, but expanding the set-up to a basket of assets leads to computational problems related to the calculation of the probability distribution of the basket price. Therefore, we circumvent this problem by employing a Hermite

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polynomial expansion matching exactly the first m moments of the model-implied basket return.

The pricing and hedging methodology we propose consists of quasi-analytical formulae: they are Black and Scholes type formulae and some of their inputs are given as the solution of a system of m equations in m unknowns. The main advantages of the new methodology are: low computational cost compared to numerical methods, especially when one prices a portfolio of options written on the same basket with different strikes and/or payoffs, since the matching procedure needs to be carried out only once; precise calculations and the availability of formulae for the Greeks. Additionally, the only prerequisite of our method is the existence of the moments of the basket and, consequently, it is applicable to the situation when some assets in the basket follow one diffusion model and other assets follow a different diffusion model.

The remaining of the article is structured as follows. Section 2 reviews the existing literature on pricing and hedging basket options. Section 3 describes the continuous-time models employed here. The new methodology is discussed in Section 4 and a numerical comparison is presented in Section 5. The final section concludes.

2. Existing contributions

The number of papers covering basket options has increased considerably in the last three decades. The available methods can be classified into analytical, purely numerical and a hybrid quasi-analytical class which is based on various expansions and moment matching techniques. Our method belongs to the last category.

By analogy to early papers on pricing Asian options, Gentle (1993) proposed pricing basket options by approximating the arithmetic weighted average with its geometrical-average counterpart so that a Black–Scholes type formula could be applied. Korn and Zeytun (2013) improved this approximation using the fact that, if the spot prices of assets in the basket are shifted by a large scalar constant C, their arithmetic and geometric means converge asymptotically. They consider log-normally distributed assets and approximate the C-shifted distribution by standard log-normal distributions. Kirk (1995) developed a technique for pricing a spread option by coupling the asset with negative weight with the strike price, considering their combination as one asset having a shifted distribution and then employing the Margrabe (1978) formula for exchanging two assets. The methods in Li et al. (2008) and Li et al. (2010) extended the procedure proposed in Kirk (1995) to the case of multi-asset spread options. Curran (1994) priced basket options with only positive weights by conditioning on the geometric basket value: the resulting formula is given as an exact term plus an approximated term. Deelstra et al. (2004, 2010) extended on Curran (1994) and obtained lower and upper bounds for the prices of basket options and Asian basket options, respectively. Similarly, Xu and Zheng (2009) derived bounds for basket options on assets following a jump–diffusion model with idiosyncratic and systematic jumps. A completely different approach has been proposed by Laurence and Wang (2004, 2005), and Hobson et al. (2005b,a). They derived model-free upper and lower bounds for basket option prices based on the prices of the European options, each on a single-asset. While the literature on pricing basket options is large, there is sparse research on calculating the hedging parameters for basket options. A notable exception is Hurd and Zhou (2010) who priced spread options and derived the Greek parameters by using fast Fourier transform under different models.

When analytical formulae are difficult to be derived under a particular model, it is common, in the finance industry, to resort to Monte Carlo methods. Control variate techniques for pricing basket options are described in Pellizzari (2001) and Korn and Zeytun (2013). While Monte Carlo methods offer a feasible solution, the computational cost may be too high even for standard-size baskets commonly traded on the financial markets. Hence, the majority of the literature on basket option pricing gravitates around approximation methods that circumvent the numerical problems generated by the high-dimensionality of basket models. Levy (1992) approximated the distribution of a basket by matching its first two moments with the moments of a log-normal density function, and then derived a Black–Scholes type pricing formula. Other works modified the log-normal approximation allowing for improved skewness and kurtosis calibration. Borovkova et al. (2007) have proposed a new methodology that can incorporate negative skewness while still retaining analytical tractability, under a shifted log-normal distribution, by considering the entire basket as one single asset.[1] This strong assumption allows the derivation of closed-form formulae for basket option pricing. On the other hand, some other research has priced basket options whose asset dynamics are more appropriate to accommodate the empirical characteristics of the asset returns. Flamouris and Giamouridis (2007) priced basket options on assets following a Bernoulli jump–diffusion process using the Edgeworth expansion; Wu et al. (2009) assumed that asset prices follow the multivariate normal inverse Gaussian model (mNIG) and employed the fast Fourier transform together with the methodology outlined by Milevsky and Posner (1998) to approximate the sum of assets following the mNIGs model as a mNIG; Xu and Zheng (2009) priced correlated local volatility jump–diffusion model deriving the Partial Integro Differential Equation (PIDE) driving the basket and approximating it via the asymptotic expansion method. Bae et al. (2011) priced basket options (with positive weights) on assets following a jump–diffusion process by using the Taylor expansion method of Ju (2002).

The technique we propose in this paper approximates the basket return at the option maturity by an Hermite polynomial expansion of a standard normal variable. This aims to solve the problems encountered by existing pricing approaches that employ polynomial expansions to approximate the probability density function of the basket values (see Dionne et al., 2006, among others). In particular, these methods provide valid approximations only for a limited set of skewness–kurtosis pairs. The main advantage of our new methodology over these previous approaches is that the matching of the moments is exact for a wider set of skewness–kurtosis set.

3. The modelling framework

From a modelling point of view, it would be more appropriate for the assets in the basket to follow models that are capable of generating negative skewness and excess kurtosis reflecting the empirical evidence in equity markets. One such flexible model is the displaced (or shifted) jump–diffusion, that is a jump–diffusion process for the displaced or shifted asset value, similar to the model discussed by Câmara et al. (2009). In the following, we define the modelling framework.

Consider the filtered probability space[2] $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0\le t\le T}, \mathbb{P})$. Let us define, on this space, the financial market consisting of the asset price processes $S^{(i)}$, $i = 1, \ldots, \Upsilon$ and the bank account $M_t = e^{rt}$ that can be used to borrow and deposit money with continuously compounded interest rate $r \ge 0$, assumed constant

1 Brigo et al. (2004) proposed a similar method to that of Borovkova et al. (2007) but their method can cope only with positive-value baskets.

2 The results in this section are proved both in Câmara et al. (2009) and in Shreve (2004, chap. 11.5). In the latter, the standard multidimensional jump–diffusion model is described and the theory can be adapted to deal with shifted assets.