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Robust loss reserving in a log-linear model

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ABSTRACT

It is well known that the presence of outlier events can overestimate or underestimate the overall reserve when using the chain-ladder method. The lack of robustness of loss reserving estimators leads to the development of this paper. The appearance of outlier events (including large claims—catastrophic events) can offset the result of the ordinary chain ladder technique and perturb the reserving estimation. Our proposal is to apply robust statistical procedures to the loss reserving estimation, which are insensitive to the occurrence of outlier events in the data. This paper considers robust log-linear and ANOVA models to the analysis of loss reserving by using different type of robust estimators, such as LAD-estimators, M-estimators, LMS-estimators, LTS-estimators, MM-estimators (with initial S-estimate) and Adaptive-estimators. Comparisons of these estimators are also presented, with application of a well known data set.

1. Introduction on loss reserving

The estimation of claim reserves and outstanding claims is very important for the operation of insurance companies, as well as for the determination of the profit. In the actuarial literature there is a variety of papers on loss reserving techniques within the chain ladder framework, namely, Kremer (1982), Taylor and Ashe (1983), Mack (1991, 1993, 1994), Renshaw (1989), Verrall (1991, 2000). The book by Taylor (2000) presents useful material on stochastic methods and practical issues, in addition to several deterministic loss reserving techniques. Furthermore, the review paper of England and Verrall (2002), and the book of Wüthrich and Merz (2008), provide a wide range of stochastic reserving methods for use in general insurance. For more sophisticated models on loss reserving, such as generalized linear mixed models, see Antonio and Beirlant (2007).

In this paper, we focus on ANOVA and log-linear models that were introduced by Kremer (1982) and used by Renshaw (1989), Verrall (1991), Zehnwirth (1985) and Christofides (1990), amongst others. Verrall (1991) considered the estimation of claims reserves and outstanding claims when a log-linear model is applied. He based his results on a general theory of estimation from linear models, as was presented by Bradu and Mundlak (1970), when the data is log-normally distributed. Christofides (1990) has shown how run-off models of the log-incremental payments can be identified and fitted in practice using multiple regression. The Log-Normal distribution has the advantage that it can be implemented without the need for specialist software. Another advantage is that other statistical techniques can also be used to allow different assumptions to be incorporated concerning the run-off pattern and the connections between origin years.

1.1. The lack of robustness of loss reserving techniques

The presence of outliers due to large claims or catastrophic events is a special problem in loss reserving calculation. Outliers can be described as points which do not follow the trend of the majority of the data. The problem appears if a trend (due to an outlier event) that appeared in one of the development years in a chain ladder setting carried on for the next years resulting in an overestimation or underestimation of claims reserves.

In particular, excess claims (large claims) lead to an unsatisfactory behavior of chain ladder methodology. Our purpose is to robustify the claims reserves calculations using robust estimators. In simple regression (two-dimensional case), it is easy to detect outlier events just by plotting the observations. This is no longer possible in the log-linear multiple regression. So, in practice, one needs a procedure that is able to lessen the impact of outliers.

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Kremer (1997) incorporated the ideas of robust statistics into loss reserving techniques by using the lagfactor-method (or link-ratio method). Verdonck et al. (2009) created a technique for detecting outlying observations in a run-off triangle of claims amounts and solved the problem of non-robustness of the chain ladder by replacing the mean by the median. Verdonck and Debruyne (2011) based on the influence function approach presented a diagnostic tool for highlighting the influence of every individual claim on the classical chain-ladder estimates. They considered the chain ladder method as generalized linear models (GLM) and obtained robust estimates of GLM in a chain ladder framework. Busse et al. (2010) designed a filter for outliers and large jumps, and presented a robust version of Mack's variance estimator. They verified the reliability of their methods with several loss triangles. Venter and Tampubolon (2010) presented an introduction of robust methods for loss reserving and compared development triangle based on the sensitivity of the reserve estimates.

In this paper a class of robust estimators is applied to a chain ladder procedure where the data is in a log linear form and that was transformed into a two-way analysis of variance. This class of estimators includes robust estimators that simultaneously attain maximum breakdown point (BP) and full asymptotic efficiency under error normality.

In our robust loss reserving estimation we initially ignore the bias present due to the robustification of the large claims, but add in a second stage, a share of the excess (correction term) to ultimate claims, to obtain a final unbiasedness. This robust log-linear regression estimation can provide quite well claims reserves estimates by guaranteeing the recovery of ultimate claims. Of course, these robust estimators can be embedded within several loss reserving techniques providing reliable claims reserves estimation.

The paper has been organized as follows. In Section 2, we present a summary of some of the most important robust estimators that appear in the statistical and actuarial literature. More specifically, we present the LAD (or L1)-estimators, M-estimators, LMS-estimators, LTS-estimators, MM-estimators (with initial S-estimate) and Adaptive-estimators, in a way that can be used to obtain robust chain ladder model estimation. In Section 3, we briefly present two models that appeared in the literature and are appropriate for the loss reserving techniques, the log-linear model of Verrall (1991) and the two way model of ANOVA by Kremer (1982). In Section 4, we provide robust estimators for Verrall's (1991) log-linear and Kremer's (1982) ANOVA loss reserving models. Numerical Illustrations are provided in Section 5 based on Taylor and Ashe (1983) data and robust estimators applied to loss reserving techniques. A comparison of robust loss reserving estimations is also provided. Finally, an overview of the results and some concluding remarks are presented in Section 6.

2. Robust inference

Very often, assumptions, made in statistics, i.e. normality, linearity, independence are at most approximations of reality. Robust regression models are useful for filtering linear relationships when the random variation in the data is not normal or when the data contain significant outliers (see Hampel et al., 1986).

In the following we present a summary of some of the most important robust estimators that appear in the statistical and actuarial literature. Some basic concepts of robust statistics are also presented in Appendix A.

2.1. LAD and M-estimators

The idea of least absolute deviation (LAD) also known as L_1 regression is actually older than that of least squares. Edgeworth in 1887 argued that outliers have a very large influence on LS because the residuals r_i are squared. The least absolute values regression estimator is determined by

$$\min_{\hat{\beta}} \sum_{i=1}^{n} |r_i|, \tag{2.1}$$

where $r_i = y_i - \boldsymbol{x}_i^T \boldsymbol{\beta}$ is the ith residual. Unfortunately the BP of L_1 regression is no better than 0%. The BP of a regression estimator is the largest proportion of the data which can be replaced by large values (outlier events) before the estimator breaks down. As its name implies, L_1 regression finds the coefficients estimate that minimizes the sum of the absolute values of the residuals.

M-estimators are generalization of maximum likelihood estimator proposed by Huber (1973), who suggested that we obtain M-estimators as solutions of the following minimization problem,

$$\min_{\hat{\beta}} \sum_{i=1}^{n} \rho\left(r_i\right), \tag{2.2}$$

where r_i is the ith residual, ρ is a symmetric function with unique minimum at zero. Differentiating this expression with respect to the regression coefficients $\hat{\boldsymbol{\beta}}$ yields, $\sum_{i=1}^{n} \psi\left(r_i\right) \boldsymbol{x}_i = 0$, where ψ is the derivative of ρ and $\boldsymbol{x}_i$ is the row vector of explanatory variables of the ith case. In practice one has to standardize the residuals by means of some estimate of S, yielding

$$\sum_{i=1}^{n} \psi\left(\frac{r_i}{S}\right) \boldsymbol{x}_i = 0, \tag{2.3}$$

where S is a scale parameter and must be estimated simultaneously. In practice, it is advisable to use $S = med\{|r_i|\}$ as an initial value.

The advantage of M-estimates is that they can be computed in much less time than other robust estimates. The disadvantage is that they are sensitive to high leverage points and they do not enjoy high breakdown point (BP). The BP of M-estimators are 0% (see Rousseeuw and Leroy, 1987, p. 145).

The location–scale M-estimators of $\boldsymbol{\beta}$, with an appropriate choice of ψ, may attain a high efficiency and at the same time be robust against large residuals. But these estimators are not robust to outliers in the design matrix space, i.e. if the explanatory variables are random or otherwise subject to errors the classical M-estimators may be unreliable. In this case the domain of the ψ function has been enlarged to include the design points, as well as the residuals.

The influence function of the Huber M-estimator (ignoring the scale) is defined as

$$IF(\boldsymbol{x}^T, y; T, F) = \frac{\psi_c(r)}{E\psi_c'(r)} (E\boldsymbol{x}\boldsymbol{x}^T)^{-1}\boldsymbol{x}. \tag{2.4}$$

The first part of the influence function in (2.4) is called the influence of the residuals and is bounded, but the second part that is called the influence of position in factor space is unbounded. Thus, a single x_i, which is an outlier in the factor space, will almost completely determine the fit. In this case the Huber estimator and all estimators defined through (2.3), including L_1, are only the first step in the robustification of the regression estimator (see Hampel et al. (1986, p. 313)).

2.2. LMS estimators and LTS estimators

A robust equivariant regression estimator that first attained the maximum asymptotic BP $= 0.5$ is the least median of squares