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The optimal insurance under disappointment theories

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ABSTRACT

In his celebrated work, Arrow (1974) was the first to discover the optimality of deductible insurance under the Expected Utility Theory; recently, Kaluszka and Okolewski (2008) extended Arrow's result by generalizing the premium constraint as a convex combination of the expected value and the supremum of an insurance indemnity, with single layer insurance as the optimal solution. Nevertheless, the Expected Utility Theory has constantly been criticized for its failure in capturing the actual human decision making, and its shortcoming motivates the recent development of behavioral economics and finance, such as the Disappointment Theory; this theory was first developed by (1) Bell (1985), and Loomes and Sugden (1986), that can successfully explain the Allais Paradox. Their theory was later enhanced to the (2) Disappointment Aversion Theory by Gul (1991), and then (3) Disappointment Theory without prior expectation by Cillo and Delquié (2006). In our present paper, we extend the problem studied by Kaluszka and Okolewski (2008) over the three mentioned disappointment models, while the solutions are still absent in the literature. We also conclude with the uniform optimality of the class of single layer indemnities in all these models.

1. Introduction

1.1. Disappointment theory

To remedy the discrepancies and inconsistencies, such as Allais Paradox (Allais, 1953) and Equity Premium Puzzle (Mehra and Prescott, 1985), between the Expected Utility Theory (EUT) and the actual decision making, various non-Expected Utility models have been proposed. For example, based on results from several variants of the original Allais' experiment, Kahneman and Tversky (1979) established the Prospect Theory (PT) to quantitatively explain the mentioned inconsistencies of the EUT with human behavior subject to certainty, common consequence, common ratio, reflection and isolation effects; Kahneman and Tversky (1992) later also enhanced the PT into the Cumulative Prospect Theory (CPT) by connecting the former with the Rank-Dependent Expected Utility Theory (RDEUT) by Quiggin (1982, 1993). Since then, non-EUT has been well received with a fruitful development that cultivates a new discipline known as *Behavioral Finance*.

In the meanwhile, Bell (1985), and Loomes and Sugden (1986) initiated an alternative direction of studies in *Behavioral Finance*, namely the Disappointment Theory (DT); decision makers experience disappointment if their 'expectation' taken prior to resolving of a lottery eventually turns out to be greater than the realized outcome. The main difference between the (C)PT and the DT is the nature of the prior reference point with respect to which loss and gain are measured. In the (C)PT, the prior reference point is arbitrarily and exogenously determined; while in the DT, the prior reference point is endogenously defined. To motivate our optimal insurance decision problem, we first give an overview on three major contemporary disappointment models in chronological order as follows.

1.1.1. Bell–Loomes and Sugden disappointment model

Bell (1985) provided the first systematic study, which was later enhanced by Loomes and Sugden (1986) that can then explain further variants of the Allais Paradox. They essentially proposed the following 'modified expected utility' to quantify human satisfaction under disappointment:

$$\mathcal{U}(X) = \mathbb{E}\left[u(X) + D(u(X) - \mathbb{E}[u(X)])\right],$$

where u is an increasing and concave function, and D is an increasing reverse S-shaped function with slope bounded above

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by 1. By using this model, Bell (1985) explained the systematic violations of independence axiom of the EUT; through it, Loomes and Sugden (1986) explained both certainty and isolation effects.

1.1.2. Gul disappointment aversion (DA) model

As the second major development of the disappointment theory, the main novelty and contribution of Gul (1991) are his intuitive explanation on the Allais Paradox that can also be consistent with his proposed weakened independence axiom. Besides, Gul suggested to decompose lotteries into elation and disappointment parts with respect to a new certainty equivalent μ; via an axiomatic approach, he also established the existence of a value function u such that the modified expected utility $\mathcal{U}$ takes the following form:

$$\mathcal{U}(X) = u(\mu(X)) = \mathbb{E}\left[u(X) + \left(\frac{1}{D} - 1\right)(u(X) - u(\mu(X)))\,\mathbb{1}_{\{X<\mu(X)\}}\right],$$

where u is an increasing and concave function, $D \in (0, 1]$ is the disappointment aversion coefficient of the decision maker, and μ is called disappointment averse certainty equivalent which is generally different from the mathematical expectation as considered in Bell (1985) and Loomes and Sugden (1986). However, the main challenge of applying his theory in practice is the implicit nature of the definition of μ, which makes any mathematics involved hard to tackle, especially over the continuum setting.

1.1.3. Cillo and Delquié disappointment model without prior expectation

Instead of assuming that the decision maker sets a single prior reference point before resolving his lottery, Cillo and Delquié (2006) initiated the third major development in the theory of disappointment by proposing a generalized model in which any physical outcomes could be a possible reference point. The rationale behind is three-fold. Firstly, it is difficult to judge whether a single prior 'expectation' is more appropriate or not, no matter it is the mathematical expectation or disappointment averse certainty equivalence. Secondly, the reference points generally do not match with the actual outcome obtained from the lottery. Thirdly, empirical studies in Ordóñez et al. (2000) showed that, for example for salary inflation, the realized outcome would trigger the disappointment feelings when compared to those outcomes better than it. In light of these, Cillo and Delquié (2006) proposed the following modified expected utility:

$$\mathcal{U}(X) = \mathbb{E}\left[u(X) - \mathbb{E}\left[D\left(u(Y) - u(X)\right)\right]\right],$$

where u is an increasing and concave function, Y is an independent lottery identically distributed as X, and D is an increasing reverse S-shaped function with slope bounded above by 1.

1.2. An overview of optimal insurance decision problems

The determination of optimal insurance arrangement has long been a popular research direction in actuarial science and insurance due to its immediate practical consequence, and constantly revisiting the problem from contemporary perspectives would often lead to a fruitful advancement in related theories and even towards pure economic theories.

Under the EUT, Arrow (1974) was the first to establish the optimality of deductible contract for an insured with respect to the expected value principle. In the game setting for risk averse agents, Borch (1975) and Raviv (1979) obtained similar results and concluded that no insurance policy with upper coverage limit is Pareto optimal. Further studies on the optimal (re)insurance problems can be found, to name a few, in the works of Cai and Wei (2013), Deprez and Gerber (1985), Kaluszka (2005), and Promislow and Young (2005). In particular, Kaluszka and Okolewski (2008) generalized Arrow's result by considering the premium principle that is a convex combination of the expectation and the (essential) supremum of the insurance indemnity, and they also concluded with the optimality of the insurance layer.

On the other hand, the theory of risk measures has recently become popular in both theory and practice in financial economics, and the promotion of use of risk measures as objective functions under various decision making problems, such as optimal insurance problems, has been well advocated. See, for instance, Balbás et al. (2009), Cai and Tan (2007), Cai et al. (2008), Centeno and Guerra (2008, 2010), Cheung (2010), Cheung et al. (2013b, 2014) and Kaluszka (2001, 2004).

Furthermore, there has been a rapidly growing body of studies on the interaction between Behavioral Finance and insurance, in particular, in connection with optimal insurance decision making under various behavioral frameworks. For instance, the optimality of insurance layer under a class of models from the CPT has been determined in Sung et al. (2011); the optimal insurance schedule under the RDEUT has been studied in Bernard et al. (2013); in Huang and Wang (2012), they determined the optimal coverage levels with respect to common insurance schedules by assuming that the insured has a S-shaped loss aversion utility while the insured would still retain the enormous part of the potential loss; in Huang et al. (2012), they investigated the optimal insurance sharing in the game-theoretic setting under the disappointment theory with respect to the best outcome proposed by Laciana and Weber (2008).

To the best of our knowledge, the determination of the optimal insurance indemnity under the mentioned DT models have still left untouched. On the other hand, aligning the Disappointment Theory with the actual decision making in financial or insurance contexts is crucial since it is natural that both the decision maker, and the insured, will feel disappointed on unfavorable financial outcomes with reference to their own preoccupation. For instance, a recent work in Cheung et al. (submitted for publication) determines the premium principle resulted under the Disappointment Aversion Theory which turns out echoing with the premium used in practice that captures the quantification of the tail risk exposure.

1.3. Organization of the paper

In this article, we shall revisit the optimal (re)insurance decision problems studied by Kaluszka and Okolewski (2008), and investigate the form of the optimal insurance indemnity under the three major disappointment theories mentioned in Section 1.1. The main contributions of our article are: (1) presenting an alternative and simpler proof for the result obtained by Kaluszka and Okolewski (2008) which is also consistent with our approaches thereafter; (2) solving for optimal insurances under various DT models which have been missed for long in the literature. In particular, we shall show that the single layer indemnity will serve as the only optimal solution under all the popular disappointment models mentioned in Section 1.1, while their corresponding objective functions show substantial difference in nature, let alone comparing with the EUT. Through a numerical example, we shall demonstrate that the optimal single layer indemnities among different disappointment models could be different.

The organization of our paper is as follows. In Section 2, the optimal insurance problem considered by Kaluszka and Okolewski (2008) will be revisited, and the alternative, shorter and more intuitive proof for their solution will be given. Each later section will be devoted to tackling the optimal insurance decision problem