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OPTIMAL RETIREMENT INCOME TONTINES

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ABSTRACT. Tontines were once a popular type of mortality-linked investment pool. They promised enormous rewards to the last survivors at the expense of those died early. And, while this design *appealed to the gambling instinct*, it is a suboptimal way to generate retirement income. Indeed, actuarially-fair life annuities making constant payments – where the insurance company is exposed to longevity risk – induce greater lifetime utility. However, tontines do not have to be structured the historical way, i.e. with a constant cash flow shared amongst a shrinking group of survivors. Moreover, insurance companies do not sell actuarially-fair life annuities, in part due to aggregate longevity risk.

We derive the tontine structure that maximizes lifetime utility. Technically speaking we solve the Euler-Lagrange equation and examine its sensitivity to (i.) the size of the tontine pool n, and (ii.) individual longevity risk aversion γ. We examine how the optimal tontine varies with γ and n, and prove some qualitative theorems about the optimal payout. Interestingly, Lorenzo de Tonti's original structure is optimal in the limit as longevity risk aversion $\gamma \to \infty$. We define the *natural tontine* as the function for which the payout declines in exact proportion to the survival probabilities, which we show is near-optimal for all γ and n. We conclude by comparing the utility of optimal tontines to the utility of loaded life annuities under reasonable demographic and economic conditions and find that the life annuity's advantage over the optimal tontine is minimal.

In sum, this paper's contribution is to (i.) rekindle a discussion about a retirement income product that has been long neglected, and (ii.) leverage economic theory as well as tools from mathematical finance to design the next generation of tontine annuities.

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