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State price densities implied from weather derivatives

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ABSTRACT

A State Price Density (SPD) is the density function of a risk neutral equivalent martingale measure for option pricing, and is indispensable for exotic option pricing and portfolio risk management. Many approaches have been proposed in the last two decades to calibrate a SPD using financial options from the bond and equity markets. Among these, non and semiparametric methods were preferred because they can avoid model mis-specification of the underlying. However, these methods usually require a large data set to achieve desired convergence properties. One faces the problem in estimation by e.g., kernel techniques that there are not enough observations locally available. For this situation, we employ a Bayesian quadrature method because it allows us to incorporate prior assumptions on the model parameters and hence avoids problems with data sparsity. It is able to compute the SPD of both call and put options simultaneously, and is particularly robust when the market faces the data sparsity issue. As illustration, we calibrate the SPD for weather derivatives, a classical example of incomplete markets with financial contracts payoffs linked to non-tradable assets, namely, weather indices. Finally, we study related weather derivatives data and the dynamics of the implied SPDs.

1. Introduction

A State Price Density (SPD) is the density function of a Risk Neutral (RN) equivalent martingale measure for option pricing, and it is a measure more tied to uncertainty than to volatility and it is indispensable for (exotic) option pricing and portfolio risk management. It does not only reflect a risk-adaptive behavior of investors based on historical assessment of the futures market, but it also gives insights about the preferences and risk aversion of a representative agent, see for example Aït-Sahalia and Lo (2000), Jackwerth and Rubinstein (1996) and Rosenberg and Engle (2002).

Consider a European call option with maturity date T and strike price K. Under the non-arbitrage principle, its price at t can be given as:

$$C(K) = e^{-r\tau} \int \max(x - K, 0) f(x) dx \tag{1}$$

where r is the risk-free interest rate, τ time to maturity and $f(x)$ is the defined SPD. The advantage of extracting the SPD directly from market prices is that volatility and other moments can easily be calculated using this SPD independent of any particular pricing model.

There are many approaches to calibrate the SPD using financial options from the bond and equity markets. Assuming a Black and Scholes (B&S) model implies that the RN measure is a lognormal distribution which may result in severe bias of the SPD estimation since certain volatility properties are not correctly reflected. As observed by Breeden and Litzenberger (1978), the SPD of any risky asset can be derived as the second derivative with respect to the strike price of an estimate of the pricing function C. A number of econometric techniques have been developed to address this calibration issue. The most notable examples include the stochastic volatility models and the GARCH models. Derman and Kani (1994), Dupire (1994) and Rubinstein (1994) implied SPDs using binomial trees, hence avoiding too strong stochasticity assumption like e.g., Geometric Brownian motion. Others like Abadir and Rockinger (2003) use hypergeometric distributions. Although useful in a variety of contexts, these (parametric) models are still susceptible to model specification.

Various non-parametric models have been employed to overcome this problem. Aït-Sahalia and Lo (1998) introduce a semi-parametric alternative where the volatility of the B&S formulation is modeled non-parametrically. From a statistical point of view, estimating the SPD becomes estimating the second derivative of a

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regression function, but the SPD needs to be a proper density function (non negative and integrates to one). This dictates that the price is decreasing and convex in terms of the strike price. How to impose these constraints presents the main difficulties of direct applications of nonparametric regression. Aït-Sahalia and Duarte (2003), Yatchew and Härdle (2006) and Härdle and Hlávka (2009) stress the importance of enforcing such shape constraints. Fan and Mancini (2009) use a non-parametric technique to estimate the state price distribution but not the density because the former is easier to estimate. Giacomini et al. (2008) use mixtures of scales and shifted t-distributions, while Yuan (2009) uses a mixture of lognormals. Curve fitting method have been presented in Rubinstein (1994) and Jackwerth and Rubinstein (1996). Liechty and Teng (2009) introduce the Bayesian quadrature model, where both the locations and weights of the support points for approximating the SPD are random variables. Most nonparametric methods require a rich body of data to achieve desired convergence properties. The main goal of this paper is to infer the SPD from markets, where trading activities are less frequently occurred.

For this purpose, we employ a Bayesian quadrature method as a calibration method for the SPD from option prices, because it allows us to incorporate prior assumptions on the model parameters and hence avoids problems with data sparsity. This approach takes a prior distribution which can be parametric (e.g. lognormal) or a uniform density. The posterior distribution of the SPD is calibrated to market data. This method is a special case of a mixture model, where the component densities are point measures.

The novelty of the Bayesian quadrature approach relies on the fact that it uses unequal weights and is in a Bayesian framework. Approximating the state price density with weighted sum of δ-functions produces good model fitting by using a parsimonious model. Bayesian inference gives a straightforward probabilistic framework and provides reasonable credible regions for the implied state price density, which can be further used for various purposes such as hedging and pricing.

We show that the proposed method has some advantages over other nonparametric methods: (1) it considers the locations and weights of the support points in the finite representation of the SPD as random variables, (2) it is parsimonious and allows for statistical inference, it enables us to construct credible regions for the current value of the SPD (3) it is computationally efficient in the sense that a Markov chain Monte Carlo algorithm with Gibbs sampler can be adopted, so that no additional tuning procedures are required for exploring the posterior distribution and (4) it is robust even if the market faces data sparsity issues. (5) These classes of Risk Neutral probabilities do not stem from market-risk-price assumptions.

We conduct our empirical analysis based on weather derivative (WD) data traded at the Chicago Mercantile Exchange (CME). WDs are newly developed financial instruments. Key features of weather derivatives are that the underlying process, i.e., temperature or rainfall index is not tradable and cannot be replicated by other risk factors (Benth et al., 2007; Härdle and López-Cabrera, 2012; López-Cabrera et al., 2013). Consequently, the Black–Scholes formula is unsuitable since an essential element of it is the tradability of the underlying. In addition, the temperature index shows apparent seasonality and it is determined by physical phenomena. An interesting feature is that weather futures and options are rarely traded and traded only at a few strike prices compared with other more frequently traded equity markets. The CME (the official WD platform) provides closing prices, which are however not the real trading prices negotiated by the market participants. The SPD enables to price options with complicated payoff functions simply by numerical integration of the payoff with respect to this density. However, data sparsity makes the SPD estimation a statistical challenge. In addition, we study the dynamics of the SPD which provides useful insight into the economic behavior of agents sensitive to weather conditions and the time inhomogeneity of the market.

This paper is structured as follows. Section 2 describes the quadrature approach and its comparison to other popular SPD density estimation methods. Section 3 conducts the empirical analysis of SPDs from CME weather option data, studies the dynamics of the SPD weather type, and gives economic interpretations from the implied SPD. In Section 4, we address the data sparsity issue by addressing why other nonparametric methods fail particularly when options with only a few strike prices are traded. Section 5 concludes the paper. All quotations of currency in this paper will be in USD and therefore we will omit the explicit notion of the currency. All the SPDs computations were carried out in Matlab version 7.6. The option data on temperature indices were obtained from CME and are also available from the research data center of the CRC 649 "Economic Risk".

2. The Bayesian quadrature method

Options are contingent claims on an underlying asset. Plain vanilla option is of either put or call type with a fixed maturity, i.e., the value of the underlying is compared to a strike price K at maturity T. Let x denote the underlying asset's price at maturity (in our application this will be equivalent to futures prices on weather indexes). For a call option, one has the payoff $\max(x-K,0)$ and for a put $\max(K-x,0)$. If we denote a put as $i=1$ and a call with $i=2$, and observed strike prices E_{ij} for $i=1,2$ and $j=1,\dots,N_i$ indexing all possible strike prices on any given day t, then the payoff function at maturity, denoted by $\wp_{ij}(x)$, can be represented by one formula,

$$\wp_{ij}(x)=(-1)^i(x-E_{ij})\mathbf{I}\left\{(-1)^i(x-E_{ij})>0\right\}(x),$$

where $\mathbf{I}\{A\}$ is an indicator function for a set A. Let t be the current time. The fair option price is given as (1) as the discounted value of the expected payoff function:

$$C_{ij}=\exp(-r\tau)\,\mathsf{E}^{Q}[\wp_{ij}(x)],$$

where $\tau=T-t$ is the time to maturity and $\mathsf{E}^{Q}[\cdot]$ is the expectation operator taken under the risk-neutral measure. The density $f(x)$ under this risk-neutral measure is the defined SPD. When the SPD $f(x)$ exists, this equals:

$$C_{ij}=\exp(-r\tau)\int\wp_{ij}(x)f(x)dx. \tag{2}$$

The left hand side of (2) is observed on the market for different payoff types depending on put/call ($i=1,2$), strike price E_{ij}, and time to maturity τ. The interest of statistical calibration is to infer the SPD $f(x)$ from a set of observed option prices.

2.1. The quadrature method

The word "quadrature" means a numerical method to approximate an integral either analytically or numerically, see Ueberhuber (1997) for example. In this research, we work the adverse way, since the interest is to infer the unknown density from the observed integrals (option prices). Define the δ-function $\delta_{\varpi}(\cdot)$ as a unit point measure at the location s by

$$\delta_s(x)=\mathbf{I}\{s=x\}.$$

The basic idea of the quadrature method is to approximate the SPD $f(x)$ by $f_N(x|w,\theta)$, a weighted sum of δ-functions:

$$f_N(x|w,\theta)=w_1\delta_{\theta_1}(x)+\cdots+w_N\delta_{\theta_N}(x), \tag{3}$$

with unknown locations $\theta=(\theta_1,\dots,\theta_N)^{\top}$ and weights $w=(w_1,\dots,w_N)^{\top}$. Here, N is a non-negative integer (smoothing)