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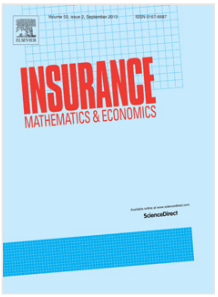

On two families of bivariate distributions with exponential marginals: Aggregation and capital allocation

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On two families of bivariate distributions with exponential marginals: aggregation and capital allocation

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Abstract

In this paper, we consider two main families of bivariate distributions with exponential marginals for a couple of random variables (X_1, X_2). More specifically, we derive closed-form expressions for the distribution of the sum $S = X_1 + X_2$, the TVaR of S and the contributions of each risk under the TVaR-based allocation rule. The first family considered is a subset of the class of bivariate combinations of exponentials, more precisely, bivariate combinations of exponentials with exponential marginals. We show that several well-known bivariate exponential distributions are special cases of this family. The second family we investigate is a subset of the class of bivariate mixed Erlang distributions, namely bivariate mixed Erlang distributions with exponential marginals. For this second class of distributions, we propose a method based on the compound geometric representation of the exponential distribution to construct bivariate mixed Erlang distributions with exponential marginals. Notably, we show that this method not only leads to Moran-Downton's bivariate exponential distribution, but also to a generalization of this bivariate distribution. Moreover, we also propose a method to construct bivariate mixed Erlang distributions with exponential marginals from any absolutely continuous bivariate distributions with exponential marginals. Inspired from Lee and Lin (2012), we show that the resulting bivariate distribution approximates the initial bivariate distribution and we highlight the advantages of such an approximation.

Keywords: Bivariate distributions; Exponential marginals; Bivariate combination of exponentials distribution with exponential marginals; Bivariate mixed Erlang distributions with exponential marginals; FGM copula; Ali-Mikhail-Haq Copula; Bladt-Nielsen's bivariate exponential distribution; Moran-Downton's bivariate exponential distribution; Aggregate claim loss; Risk measures; Capital allocation; Tail-Value-at-Risk; TVaR-based allocation rule