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Interval estimation for a measure of tail dependence

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ABSTRACT

Systemic risk concerns extreme co-movement of several financial variables, which involves characterizing tail dependence. The coefficient of tail dependence was proposed by Ledford and Tawn (1996, 1997) to distinguish asymptotic independence and asymptotic dependence. Recently a new measure based on the conditional Kendall's tau was proposed by Asimit et al. (2015) to measure the tail dependence and to distinguish asymptotic independence and asymptotic dependence. For effectively constructing a confidence interval for this new measure, this paper proposes a smooth jackknife empirical likelihood method, which does not need to estimate any additional quantities such as asymptotic variance. A simulation study shows that the proposed method has a good finite sample performance.

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1. Introduction

A recent research interest in risk management focuses on systemic risk in banking industry and insurance companies. Systemic risk concerns extreme co-movements of key financial variables. Effectively measuring tail dependence plays an important role in understanding and managing systemic risk. See Allen et al. (2012) for measuring systemic risk and using the measure to predict future economic downturns; Chen et al. (2013) for a connection of systemic risk between banks and insurers; an excellent review on systemic risk is given by Bisias et al. (2012).

Extreme co-movement usually requires measuring tail dependence of several variables. Tail dependence has been studied in the context of multivariate extreme value theory for decades. Since such a measure focuses on a far tail region of the underlying distribution, statistical inference is quite challenging due to the lack of observations. Therefore, it is always desirable to find a better measure or some competitive measures and to have an efficient inference procedure.

Suppose (X, Y) is a random vector with joint distribution F and continuous marginal distributions F_1 and F_2. Define $U = 1 - F_1(X)$ and $V = 1 - F_2(Y)$, then the distribution of (U, V) is a survival copula given by

$$C(u, v) = \mathbb{P}(1 - F_1(X) \le u, 1 - F_2(Y) \le v). \tag{1.1}$$

In order to predict an extreme co-movement of financial market, it is useful to investigate the behavior of the so-called tail copula defined as $\lim_{t\to 0} t^{-1}C(tu, tv)$, which can be employed to extrapolate data into a far tail region; see Haug et al. (2011) for an overview. When the limit is not identically zero (i.e., asymptotic dependence), one can predict rare events via estimating this limiting function. On the other hand, if the limit is identically zero (i.e., asymptotic independence), then some additional conditions are needed for predicting extreme events. To effectively distinguishing these two cases, Ledford and Tawn (1996, 1997) introduced the so-called coefficient of tail dependence $\eta \in (0, 1]$ by assuming that $C(t, t) = t^{1/\eta}s(t)$, where $s(t)$ is a slowly varying function, i.e., $\lim_{t\to 0} s(tx)/s(t) = 1$ for all $x > 0$. Therefore, η and the limit of $s(t)$ can be used to distinguish asymptotic dependence (i.e., $\eta = 1 \& \lim_{t\to 0} s(t) > 0$) and asymptotic independence (i.e., $\eta < 1$ or $\eta = 1 \& \lim_{t\to 0} s(t) = 0$). Statistical inference for η is available in Dutang et al. (2014), Draisma et al. (2004), Goegebeur and Guillou (2012) and Peng (1999).

Although copula gives a complete description of dependence among variables, having some summary measures for dependence is useful in practice. Some commonly used ones include correlation coefficient, Spearman's rho and Kendall's tau. Similarly, tail copula determines the tail dependence completely, but the coefficient of tail dependence η gives a useful measure of tail dependence. Since

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Kendall's tau is invariant to marginals and has been popular in risk management, one may wonder whether Kendall's tau can be modified to give a simple and effective measure of tail dependence as well. Recently, when the survival copula $C(u, v)$ is a bivariate regular variation, i.e., $H(u, v) = \lim_{t\to 0} C(tu, tv)/C(t, t)$ exists and is finite for $u, v \geq 0$, Asimit et al. (to appear) investigated the limit of the conditional Kendall's tau (i.e., $\theta = \lim_{u\to 0} \mathbb{E}\{sgn((U_1 - U_2)(V_1 - V_2))|\max(U_1, U_2, V_1, V_2) \leq u\}$), found that $\theta = 4\int_0^1\int_0^1 H(x, y)dH(x, y) - 1$ and showed that θ is positive for a subclass of asymptotic dependence such as elliptical tail copulas and nonpositive for a subclass of asymptotic independence such as normal copulas. Due to its ease of implementation, elliptical copulas and elliptical tail copulas have been employed in risk management; see McNeil et al. (2005). The study of tails of mixture of elliptical copulas is available in Manner and Segers (2011). A new method for constructing copulas with tail dependence is given by Li et al. (2014). Since the above measure θ involves the function H rather than some particular values of H as in η, one may expect that θ could be more effective statistically than η in distinguishing asymptotic behavior and measuring tail dependence.

For interval estimation of θ, one can estimate the complicated asymptotic variance of the proposed nonparametric estimator in Asimit et al. (to appear). In order to avoid estimating the asymptotic variance, a naive bootstrap method can be employed to construct a confidence interval, which generally performs badly in finite sample. Alternatively empirical likelihood methods have been proved to be quite effective in interval estimation and hypothesis test, which requires no estimation for any additional quantities. We refer to Owen (2001) for an overview on empirical likelihood methods. In this paper we investigate the possibility of employing an empirical likelihood method to construct a confidence interval for the limit of the conditional Kendall's tau.

We organize this paper as follows. Section 2 presents the new methodology and theoretical results. A simulation study and real data analysis on Danish fire losses are given in Section 3. All proofs are put in Section 4.

2. Methodology and theoretical results

Throughout we assume observations $(X_1, Y_1), \ldots, (X_n, Y_n)$ are independent and identically distributed with distribution function F and continuous marginals F_1 and F_2. For the study of asymptotic tail behavior of F, Asimit et al. (to appear) considered the limit of the conditional Kendall's tau, i.e., $\theta = \lim_{u\to 0} \mathbb{E}\{sgn((U_1 - U_2)(V_1 - V_2))|\max(U_1, U_2, V_1, V_2) \leq u\}$. A simple nonparametric estimator for θ is to replace the conditional expectation by its sample conditional mean, which leads to

$$\hat{\theta}(k) = \frac{\sum_{1\leq i<j\leq n} sgn((\hat{U}_i - \hat{U}_j)(\hat{V}_i - \hat{V}_j))I(\max(\hat{U}_i, \hat{U}_j, \hat{V}_i, \hat{V}_j) \leq k/n)}{\sum_{1\leq i<j\leq n} I(\max(\hat{U}_i, \hat{U}_j, \hat{V}_i, \hat{V}_j) \leq k/n)},$$

where $\hat{U}_i = 1-\hat{F}_1(X_i)$, $\hat{V}_i = 1-\hat{F}_2(Y_i)$, $\hat{F}_1(x) = n^{-1}\sum_{i=1}^n I(X_i \leq x)$, $\hat{F}_2(y) = n^{-1}\sum_{i=1}^n I(Y_i \leq y)$, $k = k(n) \to \infty$ and $k/n \to 0$ as $n \to \infty$. Under some conditions, Asimit et al. (to appear) derived the asymptotic limit of $\hat{\theta}(k)$, which has a complicated asymptotic variance. Here we investigate the possibility of employing an empirical likelihood method to construct a confidence interval without estimating the asymptotic variance explicitly. By noting that $\hat{\theta}(k)$ is a solution to the following equation

$$\sum_{1\leq i<j\leq n} \{sgn((\hat{U}_i - \hat{U}_j)(\hat{V}_i - \hat{V}_j)) - \theta\} \times I(\max(\hat{U}_i, \hat{U}_j, \hat{V}_i, \hat{V}_j) \leq k/n) = 0,$$

one may employ the empirical likelihood method based on estimating equations in Qin and Lawless (1994) to the above equation. Unfortunately such a direct application fails to achieve a chi-squared limit due to the involved U-statistic and the plug-in estimators for $U_i's$ and $V_i's$. Recently a so-called jackknife empirical likelihood method is proposed by Jing et al. (2009) to construct confidence intervals for non-linear functions including U-statistics. However, due to the involved indicator function, a direct application of the jackknife empirical likelihood function fails again to have the Wilks theorem. In order to catch the contribution made by the plug-in empirical distributions, we propose to employ the smooth jackknife empirical likelihood method proposed by Peng and Qi (2010) for constructing confidence intervals for a tail copula.

More specifically, for $l_1, l_2 = 1, \ldots, n$, define

$$\begin{cases} \hat{F}_1^{(l_1)}(x) = \frac{1}{n-1}\sum_{j=1, j\neq l_1}^n I(X_j \leq x), \qquad \hat{U}_{l_2}^{(l_1)} = 1 - \hat{F}_1^{(l_1)}(X_{l_2}), \\ \hat{F}_2^{(l_1)}(x) = \frac{1}{n-1}\sum_{j=1, j\neq l_1}^n I(Y_j \leq x), \qquad \hat{V}_{l_2}^{(l_1)} = 1 - \hat{F}_2^{(l_1)}(Y_{l_2}), \\ \hat{T}_n(\theta) = \frac{2}{n(n-1)}\sum_{1\leq i<j\leq n}\left\{sgn\left((\hat{U}_i - \hat{U}_j)(\hat{V}_i - \hat{V}_j)\right) - \theta\right\} \\ \quad \times G\left(\frac{1-\frac{n}{k}\hat{U}_i}{h}\right)G\left(\frac{1-\frac{n}{k}\hat{V}_i}{h}\right)G\left(\frac{1-\frac{n}{k}\hat{U}_j}{h}\right)G\left(\frac{1-\frac{n}{k}\hat{V}_j}{h}\right), \\ \hat{T}_n^{(l_1)}(\theta) = \frac{2}{(n-1)(n-2)} \\ \quad \times \sum_{1\leq i<j\leq n,\ i,j\neq l_1}\left\{sgn\left((\hat{U}_i^{(l_1)} - \hat{U}_j^{(l_1)})(\hat{V}_i^{(l_1)} - \hat{V}_j^{(l_1)})\right) - \theta\right\} \\ \quad \times G\left(\frac{1-\frac{n}{k}\hat{U}_i^{(l_1)}}{h}\right)G\left(\frac{1-\frac{n}{k}\hat{V}_i^{(l_1)}}{h}\right)G\left(\frac{1-\frac{n}{k}\hat{U}_j^{(l_1)}}{h}\right) \\ \quad \times G\left(\frac{1-\frac{n}{k}\hat{V}_j^{(l_1)}}{h}\right), \end{cases}$$

where $G(x) = \int_{-\infty}^x g(y)\,dy$ and g is a symmetric smooth density function with support $[-1, 1]$ and $h = h(n) > 0$ is a bandwidth. Therefore a jackknife sample is defined as

$$\hat{Z}_i(\theta) = n\hat{T}_n(\theta) - (n-1)\hat{T}_n^{(i)}(\theta) \quad \text{for } i = 1, \ldots, n.$$

Note that, in order to take care of the contributions from $\hat{U}_i's$ and $\hat{V}_i's$ in proving Wilks theorem, we do not use $G(\frac{1-\frac{n}{k}\max\{\hat{U}_i, \hat{V}_i, \hat{U}_j, \hat{V}_j\}}{h})$ instead of the product of $G's$ in the above definition of $\hat{T}_n(\theta)$. Based on this jackknife sample, a smooth jackknife empirical likelihood function for θ is obtained as

$$L(\theta) = \max\left\{\prod_{i=1}^n(np_i) : p_1 \geq 0, \ldots, p_n \geq 0 \sum_{i=1}^n p_i = 1, \sum_{i=1}^n p_i\hat{Z}_i(\theta) = 0\right\}. \tag{2.1}$$

It follows from the Lagrange multiplier technique that

$$l(\theta) := -2\log L(\theta) = 2\sum_{i=1}^n \log\left\{1 + \lambda\hat{Z}_i(\theta)\right\}, \tag{2.2}$$

where $\lambda = \lambda(\theta)$ satisfies

$$\sum_{i=1}^n \frac{\hat{Z}_i(\theta)}{1 + \lambda\hat{Z}_i(\theta)} = 0.$$

In order to show that Wilks theorem holds for the above smooth jackknife empirical likelihood method, we need some regularity

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