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Expected utility and catastrophic consumption risk[☆]

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HIGHLIGHTS

- We derive conditions on utility to avoid fragility of a cost-benefit analysis.
- The conditions ensure that expected (marginal) utility of consumption is finite.
- Our context-free results pertain to managing catastrophic consumption risk.

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ABSTRACT

An expected utility based cost-benefit analysis is, in general, fragile to distributional assumptions. We derive necessary and sufficient conditions on the utility function of consumption in the expected utility model to avoid this. The conditions ensure that expected (marginal) utility of consumption and the expected intertemporal marginal rate of substitution that trades off consumption and self-insurance remain finite, also under heavy-tailed distributional assumptions. Our results are relevant to various fields encountering catastrophic consumption risk in cost-benefit analysis.

1. Introduction

An economist, when asked to model decision making under risk or uncertainty for normative purposes, would typically work within the expected utility framework with constant relative risk aversion (that is, power utility). A statistician, on the other hand, would model economic catastrophes through probability distributions with heavy tails. Unfortunately, expected power utility is fragile with respect to heavy-tailed distributional assumptions: expected utility may fail to exist or it may imply 'incredible' conclusions.

Economists have long been aware of this tension between the expected utility paradigm and distributional assumptions (Menger, 1934), and the discussions in Arrow (1974), Ryan (1974),

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and Fishburn (1976) deal explicitly with the trade-off between the richness of the class of utility functions and the generality of the permitted distributional assumptions. Compelling examples in Geweke (2001) corroborate the fragility of the existence of expected power utility with respect to minor changes in distributional assumptions.

The combination of heavy-tailed distributions and the power utility family may not only imply infinite expected utility, but also infinite expected *marginal* utility, and hence, via the intertemporal marginal rate of substitution (the pricing kernel), lead to unacceptable conclusions in cost-benefit analyses. For example, with heavy-tailed log-consumption and power utility, the representative agent should postpone *any* unit of current consumption to mitigate future catastrophes. The latter aspect was recently emphasized by Weitzman (2009) in the context of catastrophic climate change. Weitzman also argues that attempts to avoid this unacceptable conclusion will necessarily be non-robust. Related questions about the validity of expected utility analysis in a catastrophic climate change context were analyzed by Chichilnisky (2000) and Tol (2003), and more recently by Horowitz and Lange (2009), Karp (2009), Arrow (2009), Nordhaus (2009, 2011), Pindyck (2011), Buchholz and Schymura (2012), and Chanel and Chichilnisky (2013), among others.

The current paper contributes to the literature on how to conduct an expected utility based cost-benefit analysis in the presence of catastrophic consumption risk, by deriving general theoretical compatibility results on the utility function of consumption in the expected utility model, leaving probability distributions unrestricted. Our theoretical results are context-free, and they are relevant to various fields encountering catastrophic consumption-risk analysis, such as risk management and self-insurance, finance, and environmental economics. More specifically, we obtain necessary and sufficient conditions on the utility function of consumption in the expected utility model which avoid the fragility of an expected utility based cost-benefit analysis to its distributional assumptions. These conditions ensure that expected utility and expected marginal utility of consumption – and hence the expected intertemporal marginal rate of substitution that trades off consumption and self-insurance – remain finite also under heavy-tailed distributional assumptions. Thus, they support a valid axiomatization of expected utility and avoid incredible consequences in a cost-benefit analysis.

We emphasize that this paper deals with the problem of intertemporal consumption choice and self-insurance in the presence of catastrophic consumption risk, and that its results cannot directly be translated to the setting of insurance premium calculation (Goovaerts et al., 1984; Kaas et al., 2008, Chapter 1). We expand on this later in the paper.

The remainder of the paper is organized as follows. Section 2 lists four principles on which the paper is built. Section 3 introduces the basic setting and notation. Section 4 presents the optimal consumption and self-insurance model used to conduct cost-benefit analysis. Section 5 studies expected (marginal) utility and catastrophic consumption risk within this model, deriving results on the trade-off between permitted distributional assumptions and the existence of expected (marginal) utility of consumption. Section 6 provides some examples. Section 7 generalizes the main result of Section 5 to arbitrary order of differentiation. Section 8 concludes. Proofs are relegated to Appendix.

2. Four principles

Our paper is built on four principles, which will recur in our analysis:

(i) *Catastrophic risks are important*. To study risks that can lead to catastrophe is important in many areas, e.g., financial (insurer, pension, bank, trader) distress, traffic accidents (bridge collapse, airplane crash, flight control system failure), dike bursts, killer asteroids, nuclear power plant disasters, and extreme climate change. Such low-probability high-impact events should not be ignored in cost-benefit analyses for policy making.

(ii) *Light-tailed risk may lead to heavy-tailed risk*. When x is normally distributed (light tails), then e^x has finite moments. But when x follows a Student distribution (heavy tails), then e^x has no finite moments. Light-tailed risk can easily generate heavy-tailed risk. For example, in classical Bayesian statistics, the posterior predictive distribution of an initially light-tailed distribution, often has heavy tails. A prototypical example is the case of a normal distribution, which is light-tailed if its standard deviation is known, but becomes heavy-tailed Student if uncertainty about its standard deviation is integrated out against a standard scale-invariant non-informative prior (see, e.g., Geweke, 2001; Weitzman, 2009). Even if initial processes do not have heavy tails in their distribution, this does not guarantee that consequences of these processes cannot have heavy tails. Therefore, in the presence of uncertainty, it may well be reasonable to use heavy-tailed distributional assumptions to model future (log)-consumption.

(iii) *The price to reduce catastrophic risk is finite*. Are we willing to spend all our wealth to avoid children being killed at a dangerous street corner? Or dikes to burst? Or a power plant to explode? Or a killer asteroid to hit the Earth? Or climate to change rapidly? No, we are not. To assume the opposite (that a society would be willing to offer all of its current wealth to avoid, mitigate, or self-insure against catastrophic risks) is not credible, not even from a normative perspective. There is a limit to the amount of current consumption that the representative agent is willing to give up in order to obtain one additional *certain* unit of future consumption, no matter how extreme and irreversible a catastrophic risk may be. In other words: the expected pricing kernel is finite.

(iv) *A good model 'in the center' is not necessarily good 'at the edges'*. Suppose we have estimated a function $C = a + bY$, relating consumption C to disposable income Y. The dots in Fig. 1 represent the data and the line gives the resulting least-squares prediction $\hat{C} = \hat{a} + \hat{b}Y$. For incomes in the center, roughly between 40 and 80, the consumption function can be well approximated by the regression line. How useful is this result for very low (or very high) incomes? Not very useful. For very low incomes, predicted consumption would even become negative! This does not mean that a linear consumption function is useless, but it is only useful in the center of the domain. Models are approximations, not truths (cf. Goovaerts et al., 2010, p. 301), and approximations may not work well if we move too far away from the point of approximation. Examples are abundant and easy to find. Newton's theory works fine for cars and trains, but not for space ships. Pharmaceutical testing is typically performed on adult men, and may (and often does) work differently for women and children (Litt, 1997). In quantitative risk management, it has become common practice to use separate models for the central part of the data and for the extremes, and to 'glue' the models together at a carefully chosen order statistic; see Peng (2001), Johansson (2003), and Necir and Meraghni (2009), and the references therein.

In what follows we accommodate principles (i) and (ii) by leaving distributional assumptions unrestricted. We account for (iii) by ensuring that expected (marginal) utility of consumption remains finite. Our necessary and sufficient conditions (to be presented shortly) imply that the widely adopted power utility function should not be used with catastrophic (heavy-tailed) consumption