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Modeling mortality and pricing life annuities with Lévy processes

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HIGHLIGHTS

- Poisson GLM with stochastic force of mortality is defined using α-stable Lévy subordinators.
- GLM is used to estimated the parameters.
- Model comparison is done.
- Mortality rates are predicted.
- Life annuities are priced and compared.

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ABSTRACT

We consider the pricing of annuity-due under stochastic force of mortality. Similarly to Renshaw et al. (1996) and Sithole et al. (2000), the force of mortality will be defined using an exponential function of Legendre polynomials. We extend the approach of Ballotta and Haberman (2006) by conditionally adding α-stable Lévy subordinators in the force of mortality. In particular, we focus on the Gamma and Variance-Gamma processes in order to show how Lévy subordinators can capture mortality shocks. Generalized Linear Models is used to estimate coefficients of the explanatory variables and the Lévy process. For this purpose, the coefficients of the process are obtained by maximizing the log-likelihood function. We use the mortality data of males in Japan from 1998–2011 and the U.S. from 1965–2010 in order to compare our results with the model proposed by Renshaw et al. (1996). Some preferences are indicated based on Akaike's information criterion, Bayesian information criterion, likelihood ratio test and Akaike weights to support the proposed model. We then use a cubic smoothing spline method to fit the interest rate curve and illustrate some over (under) estimations in the prices of annuities under the structure suggested by Renshaw et al. (1996).

1. Introduction

One of the most important tasks for actuaries is to evaluate insurance products and liabilities. These evaluations for life insurance products should be based on modeling mortality rate since the policyholder's benefits directly depend on survival and/or death. The pricing can be done without any problem, in the world of certainty. However, in the real world, actuaries are mainly confronted with three different source of risks: investment risks, mortality risks and the risk of fluctuation in the interest rates.

Before introducing our methodology, we briefly review some of the well known approaches proposed to model mortality improvements. These include the Lee–Carter (LC) model and its generalizations (Lee and Carter, 1992; Renshaw and Haberman, 2003, 2006; Lee and Miller, 2001; Jong and Tickle, 2006); two-factor model for stochastic mortality (Cairns et al., 2006); P-splines model (Currie et al., 2004; Currie, 2006).

Recently Lévy process has been considered to model mortality rates. Biffis (2005) proposes affine jump diffusion processes to model asset prices and mortality dynamics. However, this model cannot guarantee a nonnegative force of mortality as pointed out by Chen and Cox (2009) where they include a jump process into the original Lee Carter model. Moreover, they apply their proposed model to forecast mortality rates and examine mortality securitization. Hainaut and Devolder (2008) consider the force of mortality as a combination of deterministic and stochastic component. Wang et al. (2011) claim that mortality indices in the LC model have tails thicker than a normal distribution and appear to be skewed. Therefore they suggest two infinitely divisible

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distributions, Generalized Hyperbolic and Classical Tempered Stable distributions to model the mortality indices in the LC model.

Our contribution in this paper is to introduce a random shock in the force of mortality. Ballotta and Haberman (2006) consider modeling the force of mortality with a Brownian motion as the perturbation part. We generalized this approach by including a Lévy subordinator to take into account any mortality shocks. For the sake of estimation of the parameters, we consider a Gamma process as well as a Variance-Gamma process and follow the generalized linear model proposed by Renshaw et al. (1996) and Sithole et al. (2000). The log-likelihood function is maximized to estimate the coefficients of the perturbation part. We consider two different mortality data sets including Japan and the US For Japan and based on the data over a short period of time, we find that the model proposed by Renshaw et al. (1996) can over evaluated the liabilities. Also, our analysis shows that for the US and with the mortality data over a long period of time, the price of whole life annuities can be under-estimated based on the modeling structure in Renshaw et al. (1996).

The paper is organized as follows: In Section 2, we specify the proposed model and the data used. For the estimation purposes, we give the analytical details of the suggested model for the Gamma and Variance-Gamma processes in Section 3. The fitting methodology and projections for Japan and the US are explained in Section 4. We compare our proposed models with Renshaw et al. (1996) in Section 5. In Section 6, we analyze the effect of evolution of the long term interest rate and price a whole life annuities due by using the projected values under the proposed models. Also we carry out a simulation study to assess the tail distribution of the prices in this section. Finally, Section 7 gives the conclusion of this study.

2. Proposed model

The GLMs have been used in actuarial applications since the early 1980s, see McCullagh and Nelder (1989). They proposed to fit GLM's to different data sets, including motor insurance and marine insurance. The GLM can also be used to model the force of mortality, i.e. the instantaneous rate of mortality at a certain age and a calendar year. Renshaw et al. (1996) propose a Poisson GLM to model the force of mortality, $\mu(x,t)$, at age x in year t with a log link function that is:

$$\mu(x,t) = \exp\Big[\beta_0 + \sum_{j=1}^{s}\beta_j L_j(x') + \sum_{i=1}^{r}\alpha_i t'^i + \sum_{i=1}^{k}\sum_{j=1}^{l}\gamma_{ij}L_j(x')t'^i\Big],$$
$$x\in[x_1,x_2],\; t\in[y_1,y_2], \qquad (2.1)$$

subject to the convention that some of the γ_{ij} terms may be pre-set to 0. Here $L_i(x')$ are Legendre polynomials generated by

$$L_0(x) = 1, \qquad L_1(x) = x,$$
$$(n+1)L_{n+1}(x) = (2n+1)xL_n(x) - nL_{n-1}(x), \quad n \ge 1.$$

The unknown parameters α_i, β_j, γ_{ij} need to be estimated. Also x' and t' are the transformed ages and transformed calendar years that map x and t onto the interval $[-1,1]$, as defined by:

$$x' = \frac{2x - \max(x) - \min(x)}{\max(x)-\min(x)}, \qquad t' = \frac{2t - \max(t) - \min(t)}{\max(t)-\min(t)}.$$

The number of terms in the model for each summation in (2.1) can be determined by monitoring improvements of residual deviances due to adding a new term. Let $D(x,t)$ and $\mu(x,t)$ be the actual number of deaths and the force of mortality occurring at aged x and in calendar year t, respectively. We considered $D(x,t)$ as a Poisson response random variable of a generalized linear model with mean and variance given by

$$\mathbb{E}[D(x,t)] = m(x,t) = r(x,t)\mu(x,t),$$
$$\mathbb{V}ar[D(x,t)] = \phi m(x,t),$$

where $r(x,t)$ is a non-random constant equal to the number of people exposed to the risk of death and ϕ is a scale parameter.

2.1. A new stochastic mortality model

Let $Y_{t'}$ be a mean reverting process defined by:

$$\begin{cases} dY_{t'} = -aY_{t'}d_{t'} + \gamma dZ_{t'}, & t' > -1, \\ Y_{-1} = 0 \end{cases} \qquad (2.2)$$

where a and γ are unknown constants and $(Z_{t'} : t' \ge -1)$ is a time-changed Lévy process with an initial value $Z_{-1} = 0$. In order to incorporate a perturbation in the force of mortality, we assume that the conditional expected value of $D(x,t)$, given an unobservable random variable $Y_{t'}$, is

$$\mathbb{E}[D(x,t) \mid Y_{t'}] = r(x,t)\exp\Big[\beta_0 + \sum_{j=1}^{s}\beta_j L_j(x') + \sum_{i=1}^{r}\alpha_i t'^i + \sum_{i=1}^{k}\sum_{j=1}^{l}\gamma_{ij}L_j(x')t'^i + \delta_1 Y_{t'}\Big],$$
$$x\in[x_1,x_2],\; t\in[y_1,y_2]. \qquad (2.3)$$

The mean reverting property means that the process goes towards its long-term mean. This maybe consider as a desirable property when modeling mortality. A Lévy process has independent, stationary increments and is continuous in probability. See Applebaum (2004) for an introduction to Lévy processes. Lévy Ito decomposition states that Z_t can be decomposed into three components: one deterministic drift, one Brownian motion and one jump process. A Levy process with increasing trajectories is called subordinator, and therefore it does not have any Brownian component. In this paper, to estimate the parameters in (2.3), we first consider a special case of Lévy subordinator for Z_t in (2.2) namely, Gamma process. Next, we allow both negative and positive jumps by including the Variance-Gamma process. As a result, we can compare these stochastic models with the Renshaw's model in (2.1), in which the force of mortality contains no stochastic shock.

2.2. Data

We use mortality rates for Japan, males, 1998–2011, ages 50–100 as well as the US, males, 1965–2010, ages 50–80. The data sets are available at Human Mortality Database.[1] Fig. 1 shows the crude mortality rates in Japan for some selected ages from 1998 to 2011. In this plot, it is clear that in some particular calendar years, the observed mortality rates have had positive or negative fluctuations. For example, there is a dramatic increase in the mortality rates for age 52 during 1998–2011. To visualize this positive jump clearly, we use a box plot for the differences in mortality rates of each consecutive year, as shown in Fig. 2. The solid black segments indicate the median of the differences, while the lower and upper limits of the box represent the lower quartile (Q_1) and the upper quartile (Q_3), respectively. The upper (lower) whisker is drawn at the most extreme differences which is less (greater) than or equal to the $Q_3 + 1.5 \times IQ$ ($Q_1 - 1.5 \times IQ$), where $IQ = Q_3 - Q_1$ is the inter-quartile. Any difference outside these whiskers can be considered as a possible outlier and is indicated

1 www.mortality.org.