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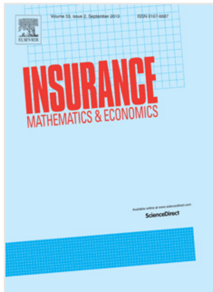

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On the convex transform and right-spread orders of smallest claim amounts

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Abstract

Suppose $X_{\lambda_1}, \cdots, X_{\lambda_n}$ is a set of Weibull random variables with shape parameter $\alpha > 0$, scale parameter $\lambda_i > 0$ for $i = 1, \cdots, n$ and $I_{p_1}, \cdots, I_{p_n}$ are independent Bernoulli random variables, independent of the X_{λ_i}'s, with $E(I_{p_i}) = p_i$, $i = 1, \cdots, n$. Let $Y_i = X_{\lambda_i} I_{p_i}$, for $i = 1, \cdots, n$. In particular, in actuarial science, it corresponds to the claim amount in a portfolio of risks. In this paper, under certain conditions, we discuss stochastic comparison between the smallest claim amounts in the sense of the right-spread order. Moreover, while comparing these two smallest claim amounts, we show that the right-spread order and the increasing convex orders are equivalent. Finally, we obtain the results concerning the convex transform order between the smallest claim amounts and find a lower and upper bound for the coefficient of variation. The results established here extend some well-known results in the literature.

Keywords: Smallest Claim Amount; Convex Transform Order; Right-Spread Order; Increasing Convex Order; Wiebull Distribution; Coefficient of Variations.

1 Introduction

It is quite important for an actuary to be able to express preferences between random future gains or losses. For this purpose, stochastic ordering results become very useful. Stochastic orders have been used in various areas including management science, financial economics, insurance, actuarial science, operations research, reliability theory, queuing theory and survival analysis. Interested readers may refer to Müller and Stoyan (2002) and Shaked and Shanthikumar (2007) for comprehensive discussions on univariate and multivariate stochastic orders.

Annual premium is the amount paid the policyholder on an annual basis to cover the cost of the insurance policy being purchased. Indeed, it is the primary cost to the policyholder of transferring the risk to the insurer which depend on the type of insurance (life, health, auto, etc). It is of interest to note that smallest claim amounts can have a critical role in insurance for determining annual premium. This reason can be one of the motivations our work.

The Weibull distribution is one of the commonly used distributions in reliability, life testing and actuarial science. A random variable X is said to have the Weibull distribution with shape parameter $\alpha > 0$ and scale parameter $\lambda > 0$ (denoted by $X \sim W(\alpha, \lambda)$) if its probability density function is given by

$$f(x; \alpha, \lambda) = \alpha\, \lambda^{\alpha}\, x^{\alpha-1}\, e^{-(\lambda\, x)^{\alpha}}, \qquad x > 0,\ \alpha > 0,\ \lambda > 0.$$

It is well-known that the hazard rate of Weibull distribution is decreasing for $\alpha < 1$, constant for $\alpha = 1$, and increasing when $\alpha > 1$. One may refer to Johnson et al. (1994) and Murthy et al. (2004) for comprehensive discussions on various properties and applications of the Weibull distribution.