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Jump diffusion transition intensities in life insurance and disability annuity

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ABSTRACT

We study the effects of jump diffusion transition intensities on a life insurance and disability annuity. To do so, we use a multi-states Markov chain with multiple decrement. Assuming independent statewise intensities, we evaluate the prospective reserve for this scheme where the insured life is in Active or Disabled state at inception, respectively. We also examine the components of the prospective reserves by changing the relevant parameters of the transition intensities, which are the jump size, the average frequency of jumps as well as the diffusion parameters, assuming deterministic rate of interest. The computation of the reserve sensitivity with their figures are provided.

1. Introduction

Recent trends in mortality and morbidity imply that it is not only important to be able to mitigate mortality and longevity risks, but also the risks related to the time spent in the disability state. In short, living longer is pretty much meaningless if the remaining life is spent with limited ability to perform activities of daily living (ADL). Severe disability has increased in Japan, a country where living beyond 110 years is not impassable, as well as in Australia during the period of 1990s. Over the period of 1980s and 1990s, physical functional limitations at old age showed an increasing trend in countries such as New Zealand, Australia, Canada, Taiwan as well as Great Britain; decreasing trend in the United States and appear to stagnate in the Netherlands (see Robine and Michel (2004) and references therein).

An Australian life expectancy survey ranging from 1998 to 2009 revealed that while older Australians are living longer without severe limitation in basic daily activities, the ageing of the population and the increasing longevity are leading a greater number of older people with disability and severe activity limitation (Wen, 2012). In handling the issues of disability transition, it is important to consider not only the decrease in mortality but also the potential increase or decrease in disability.

Previously, many studies focused on the uncertainty and randomness of mortality intensity, and thereby suggesting the use of stochastic mortality model (refer to Marocco and Pitacco (1998), Milevsky and Promislow (2001), Dahl (2004), Biffis (2005) and Cairns et al. (2006), for instance). Modelling mortality intensity as a stochastic process is useful in obtaining more realistic premiums and reserves, as well as in mitigating the systematic risk of an insurance portfolio related to prolonged longevity via hedging method (Dahl and Møller, 2006). One possible attempt to resolve the issue of uncertainty in mortality projection is by issuing mortality and longevity securities. Under this approach, insurance companies and pension funds attempt to hedge their longevity and catastrophic mortality risks by subscribing to longevity and mortality securities circa 2000–2005. However these securities lack market liquidity on top of requiring massive upfront capital, potentially leaving no capital to hedge other risks (see Cairns et al. (2005)). Many literature have also attempted to formulate the pricing of these securities (Milevsky and Promislow (2001), Blake et al. (2006), Wills and Sherris (2010) and Chen et al. (2013) for instance).

Dahl (2004) exploited the tractability of affine processes and model the mortality intensity by a general diffusion model. The process was then used to derive the value of sum insured at time t, which depends on the development of underlying mortality intensity. The continuous affine process was then extended in Biffis (2005) by including the jump component, whereby the illustrations utilized a jump diffusion model that was allowed to take negative values, as well as a bivariate square root diffusion model. Recently, the continuous affine processes in Dahl (2004)

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was then applied to evaluate life insurance liabilities represented by multistate Markov chain in Buchardt (2014).

The jump diffusion process that has been used to represent variables such as the default intensity, asset returns as well as interest rate (such as the work by Duffie and Garleanu (2001), Filipović (2001), Das (2002), Brigo and El-Bachir (2006), Mortensen (2006), Kou (2008), Brigo and El-Bachir (2010), Mendoza-Arriaga and Linetsky (2014) and Mohd Ramli and Jang (2015)) allows us to capture the effects of shocks, while general diffusion models are not able to capture jumps in these variables. Hence modelling mortality and morbidity rate as a jump diffusion process is required and viable due to the possibility that chronic illnesses coming as a shock event, thereby causing functional limitations as well as disability (such as stroke caused paralysis, heart attack, pandemic disease, and disability caused by radiation exposure). It is also of practical interest for public and private sector in health insurance to consider modelling both the mortality rate and the morbidity rate to evaluate the reserve of a term insurance and disability annuity scheme.

Norberg (1995) presented multi-state life/disability insurance models with a comparative study of stochastic interest and stochastic mortality. The applications and modelling of multistate models in health insurance to obtain safety margins on premiums and reserves can be found in Christiansen (2012, 2013). Recently Buchardt (2014) showed how to valuate life insurance liabilities using affine processes for modelling dependent interest and transition rates with a multi-state Markov chain.

In this study we attempt to extend the jump diffusion framework in Biffis (2005) to represent the mortality and morbidity rates of a 3-state and 2-state hierarchical Markov chains. Unlike Fong et al. (2013) whom considered a Markov chain with transient states and an absorbing state, we adopt the hierarchical Markov chain and disregard the possibility of recovery as in Buchardt (2014) and Olivieri and Ferri (2003) as we assume that at the age of 65, the elderlies are the group most at risk of losing physical functioning that would result in permanent physical impairment. Even after a recovery, disabilities would most likely cause the elderlies not to be entirely fit to re-join the employment pool and gain income as when they were younger, thereby making annuity an important element in a long term care (LTC) insurance scheme. Hence, by the chronic characteristics of the long term care illness, the assumption of permanent disability is justifiable. Disregarding the uncertainty of financial risk element by letting the interest rate to be deterministic, we then apply the jump diffusion process in hierarchical Markov chain to model the random expansion and compression of morbidity, and use it to evaluate the prospective reserve of a term insurance and disability annuity scheme.

This article is structured as follows: we describe the jump diffusion process used to represent the statewise transition intensities in Section 2, together with the derivation of the multivariate joint Laplace transform. This explicit expression for the joint Laplace transform of the time integral of the jump diffusion process provides a closed-form solution for the joint survival/non-disabled probability that essentially coincides with the expression for the bond price in interest rate model (Jang, 2007). This also leads to model default risk and price for a wide range of credit-sensitive instruments such as CDS rate (Mohd Ramli and Jang, 2015). To the best of our knowledge, modelling both the mortality rate and the morbidity rate via the joint Laplace transform of a multivariate jump diffusion process is the first contribution in the context of evaluating the prospective reserve for the insured life.

It is then followed by Section 3 consisting of the life insurance and disability annuity model setup, entailing the probabilistic setup to describe the model framework. Section 4 shows how we apply the jump diffusion model to a term insurance and disability annuity, using an example whereby the statewise intensities are independent. Assuming deterministic rate of interest, we evaluate the prospective reserve for this scheme where the insured life is in Active or Disabled state at inception, respectively. We then examine the sensitivity of prospective reserve components by changing the relevant parameters of the transition intensities, which are the jump size, the average frequency of jumps as well as the diffusion parameters, with their figure illustrations. Section 5 concludes the study.

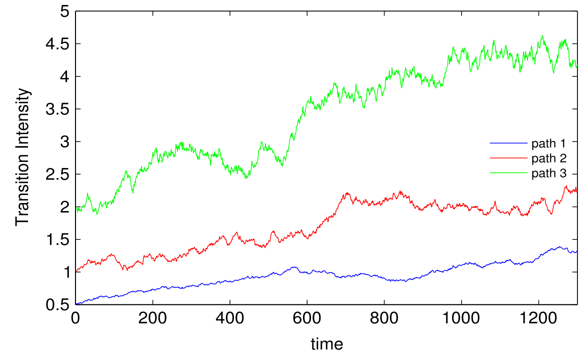

Fig. 1. Three simulated paths of jump diffusion process.

2. Jump diffusion processes

In this section we derive the joint Laplace transform of a multivariate jump diffusion process, conditional on the history of its evolution up to time t described by a filtration $\mathcal{G}_t = \sigma\left\{\left(\lambda^{(1)}(s), \ldots, \lambda^{(n)}(s)\right) : s \le t\right\} \in \mathbb{R}^n$. The transition rates from state i, $\lambda^{(i)}(t)$ is assumed to follow a jump diffusion process as defined below:

$$\begin{aligned} d\lambda^{(i)}(t) &= c^{(i)}\left(b^{(i)} + a^{(i)}\lambda^{(i)}(t)\right)dt \\ &\quad + \sigma^{(i)}\sqrt{\lambda^{(i)}(t)}dW^{(i)}(t) + dL^{(i)}(t), \end{aligned}$$

$$L^{(i)}(t) = \sum_{h=1}^{M(t)} Y_h^{(i)}, \tag{1}$$

where $c^{(i)}b^{(i)}$ represents the long term mean level of the factor being modelled (state-wise transition intensity), $c^{(i)}a^{(i)}$ represents the drift coefficient, which is the speed at which the factor is driven back to its long term mean with $c^{(i)}a^{(i)} < 0$, $\sigma^{(i)}$ is the diffusion coefficient and $W^{(i)}(t)$ is a standard Brownian motion governing the process. Fig. 1 exhibits MATLAB illustration of the simulated paths of three jump diffusion process.

We define as a pure jump process with $M(t)$ being the number of jumps, representing total number of events up to time t and $Y_h, h = 1, 2, \ldots, M(t)$ are their sizes. The point process $M(t)$ is independent of the vector sequence of jump sizes. Y_h's occurrence is assumed to be simultaneous for all i and that they are independent and identically distributed (i.i.d) with distribution function $F(y)$.

In this model, the dependence between the intensities $\lambda^{(i)}(t)$ comes from the common event arrival process $M(t)$. We assume that event arrival process $M(t)$ follows a homogeneous Poisson process with frequency parameter ρ. We impose dependency between the jump sizes of each intensity using a multivariate distribution.

We also define the integrated transition intensities $\Lambda^{(i)}(t) = \int_0^t \lambda^{(i)}(s)\,ds$ for the purpose of derivation of the joint survival/non-disabled probabilities in the next section.