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Optimal assets allocation and benefit outgo policies of DC pension plan with compulsory conversion claims

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HIGHLIGHTS

- Study optimal asset allocation and benefit outgo policies of DC pension plan.
- Derive closed-forms of the optimal policies under a new criterion.
- The continuous-time risk model associated with the problem is initially introduced.
- Economic behaviors of the optimal policies are analyzed by MCM.

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ABSTRACT

In this paper, we study optimal asset allocation and benefit outgo policies of DC (defined contribution) pension plan. We extend He and Liang model (2013a,b) to describe dynamics of individual fund scale during distribution period. The fund scale is affected by investment return, benefit outgo and mortality credit. The management of the pension plan controls the asset allocation and benefit outgo policies to achieve the objective of pension members. The goal of the management is to minimize accumulated deviations between the actual benefit outgo and a pre-set target during the whole distribution period. The performance function (criterion) is the weighted average of the square and linear deviations to express more penalty on negative deviation than positive deviation. Using HJB (Hamilton–Jacobi–Bellman) equations and variational inequality methods, the closed-forms of the optimal policies are derived. The counterintuitive effect of the optimal proportion allocated in the risky asset with respect to the fund scale is also derived, and the optimal benefit outgo has the form of the spread method. Moreover, we use Monte Carlo Methods (MCM) to analyze economic behaviors of the optimal asset allocation and benefit outgo policies.

1. Introduction

In this paper, we study optimal asset allocation and benefit outgo policies of DC (defined contribution) pension plan with compulsory conversion claims. Before conversion time, the pension fund is allowed to be invested in a risk-free asset and a risky asset, and the management of the fund chooses the asset allocation policies to achieve the objectives of pension members.

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The optimal control theory has been extensively applied in the asset allocation problems of the pension plan. Cairns (2000) and Josa-Fombellida and Rincón-Zapatero (2004) assume that the risky asset follows geometric Brownian motions and study the optimal asset allocation policies. The results show that the optimal asset allocation policy is counterintuitive, i.e., the proportion allocated in the risky asset increases with respect to the decrease of the fund scale, and vice versa.

In addition to the asset allocation policy, the benefit outgo policy is the other important control variable for the management. In recent years, one seldom studies the optimal control problem of the DC pension plan from the individual member's perspective. Most literatures, based on the total members' perspective, study the optimal asset allocation and contribution policies of DB (defined benefit) pension plan to minimize solvency risk and contribution risk. Josa-Fombellida and Rincón-Zapatero (2001) and Ngwira and Gerrard (2007) study the optimal asset allocation and contribution policies of the DB pension plan, and the optimal contribution policy explains the spread effects, i.e., the optimal contribution rate is the pre-set target multiplied by the spread.

In order to deeply investigate the optimal control problem of the DC pension plan from the individual member's perspective, we extend He and Liang model (2013a,b) to describe the dynamics of the fund scale before the conversion time. This is the first time to establish the continuous time stochastic differential equation that the fund scale dynamics satisfy during the distribution period by approximating the discrete time model in Blake et al. (2003). In this model, the dynamics of the fund scale are mainly affected by the investment return, benefit outgo and the mortality credit. For simplicity, we choose the De Moivre Model (cf. Kohler and Kohler, 2000) to characterize the force of mortality function.

In order to secure the old-care utility at the higher age, most DC pension plans have compulsory conversion claims, i.e., the fund is fully allocated in the risk-free asset and is converted into annuities at the conversion time. Horneff et al. (2008) study the optimal conversion time problem in the discrete time framework. In the model, the authors simulate the benefit outgo performances and compare the old-care utilities between the pension plans with different conversion times. The results state that the longer conversion time will enhance the utility of the pension member and reduce the security. But we find that it is difficult to treat conversion time as a control variable by the limitation of the stochastic control methods. Milevsky and Young (2007) control the conversion time to achieve the CRRA (constant relative risk aversion) utility of the pension member. This problem is well solved by the multi-plicate separable characteristics of the CRRA utility. In this paper, the conversion time is a predetermined exogenous variable and we choose the optimal one by numerical analysis.

For the performance criterion aspects, the pension member and the management form the principle-agent relationship. The management of the pension plan should choose appropriate control policies to maximize the old-care utility of the pension member. The objectives of the management are divided into two categories: maximization of the CRRA and CARA (constant absolute risk aversion) utilities of the benefit outgo, and minimization of the deviations between the actual benefit outgo and a pre-set target. The literatures on the former objectives include Battocchion and Menoncin (2004) and Milevsky and Young (2007). They choose power utility and exponential utility of the benefit outgo as the performance criterions, respectively. The idea of the latter objective originates from the work of Cairns (2000). The widely recognized performance criterion in the optimal pension management problem is to minimize the fluctuation risk of the benefit outgo. The common approach to measuring the risk is to consider the deviation between the actual benefit outgo and a pre-set target. In order to penalize more on the negative deviation than the positive deviation, the square and linear deviations are both contained in the performance function, we refer the interested readers to Chang et al. (2003).

Using HJB equations and variational inequality methods and similar procedures in Ngwira and Gerrard (2007), we solve the optimal stochastic control problem and derive the closed-forms of the optimal asset allocation and benefit outgo policies. The results of Lions and Sznitman (1984) guarantee the dynamics of the fund scale are uniquely determined by the stochastic differential equation with respect to the optimal feedback functions. Furthermore, we use MCM to investigate the impacts of the conversion time and the penalty degree for the negative deviation on the optimal asset allocation and benefit outgo policies.

The rest of this paper will be organized as follows. In Section 2, we extend He and Liang model (2013a,b) to describe the dynamics of the fund scale before the conversion time. The management chooses the proportion allocated in the risky asset and the benefit outgo as control variables. By modeling the investment return, benefit outgo and the mortality credit, we establish the stochastic differential equation that the dynamics of the fund scale satisfy. We choose minimizing weighted average of the square and linear deviations between the actual benefit outgo and the pre-set target as a criterion. In Section 3, using HJB equations and variational inequality methods, we get the closed-forms of optimal asset allocation and benefit outgo policies. We also investigate the impacts of the conversion time and the penalty degree for the negative deviation on the optimal policies by numerical analysis in Section 4. The conclusions of this paper are given in Section 5.

2. The stochastic optimal control problem

In this paper, we study the optimal asset allocation and benefit outgo policies of the DC pension plan with compulsory conversion claims. In order to secure the old-care utility at the higher age, most pension plans have compulsory conversion claims. Before the conversion time, the fund is allowed to be invested in a risk-free asset and a risky asset. The management of the fund dynamically chooses the proportion allocated in the risky asset and the benefit outgo to achieve the objectives. After the conversion time, the fund is compulsorily converted into annuities and the fund is fully invested in the risk-free asset.

We extend He and Liang model (2013a,b) to describe the dynamics of the fund scale during the distribution period, which is the first time to establish the stochastic differential equation that the dynamics of the fund scale satisfy based on the individual pension member's perspective.

From the individual member's perspective, we model the dynamics of the individual member's fund scale precisely by actuarial principles. The change of the fund scale is generated by the investment return, benefit outgo and the mortality credit. In the DC pension plan, the benefit outgo is not predetermined and it is affected by the fund scale of the individual account. We seldom study the DC pension management problem from the total members' perspective. Furthermore, it is easy to estimate the expected benefit outgo of the individual member. From the total members' perspective, which is often used in the DB (Defined Benefit) pension management, their funds are accumulated in a single account and are invested and distributed as a whole. As the scale and the structure of the members at different ages are dynamical year to year, it needs strong assumptions and approximations to estimate the expected benefit outgo and the expected fund scale of the total members.

Denote

$$\Delta\delta_t^{\frac{1}{n}} = \pi \frac{S^1_{t+\frac{1}{n}} - S^1_t}{S^1_t} + (1-\pi)\frac{S^0_{t+\frac{1}{n}} - S^0_t}{S^0_t},$$