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Statistical financial reporting for variable annuity guaranteed death benefits: Market practice, mathematical modeling and computation

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ABSTRACT

As more regulatory reporting requirements for equity-linked insurance move towards dependence on stochastic approaches, insurance companies are experiencing increasing difficulty with detailed forecasting and more accurate risk assessment based on Monte Carlo simulations. While there is vast literature on pricing and valuations of various equity-linked insurance products, very few have focused on the challenges of financial reporting for regulatory requirement and internal risk management. Most insurers use either simulation-based spreadsheet calculations or employ third-party vendor software packages. We intend to use a basic variable annuity death benefit as a model example to decipher the common mathematical structure of US statutory financial reporting. We shall demonstrate that alternative deterministic algorithms such as partial differential equation (PDE) methods can also be used in financial reporting, and that a fully quantified model allows us to compare alternatives of risk metrics for financial reporting.

1. Background

The variable annuity (VA) guarantee product is arguably the most complex investment-combined insurance product available to individual investors. Since the introduction of investment guarantees to the market in late 1990s, product features have become more and more sophisticated and made it increasingly difficult to quantify and assess the investment and longevity risks embedded in these variable annuity riders.

All insurers authorized to do business in the United States are required to prepare statutory financial statements in accordance with accounting principles established by the National Association of Insurance Commissioners (NAIC), known as the Statutory Accounting Principles. The American Academy of Actuaries (AAA) is an advisory body representing practicing actuaries in the US, which often makes recommendations of regulatory standards that are later adopted by the NAIC. There are two main components of statutory financial reporting for variable annuities that have seen drastic changes of methodology in the recent few decades.

1. Principles-based reserving (PBR)
   Reserve calculation is a standard practice of life insurers for setting aside a certain amount of liquid assets in order to cover claims from in-force insurance policies. Since the early dates of the life insurance business in the 1800s, insurers have always used a formula-based static approach to calculate reserves. However, with the convergence of insurance and capital markets in past few decades, investment-combined insurance products have grown in complexity, which led to the need for a new method for calculating life insurance policy reserves to account for market risks. This new method, PBR, establishes principles upon which reserves are based, rather than specific formulas. It requires insurers to hold reserves that consider a wide range of future economic conditions that closely reflect true risk profiles of their products. In 2008, the NAIC adopted the Variable Annuity Commissioner's Annuity Reserve Valuation Method (VA-CARVM), known as the Actuarial Guideline XLIII (AG-43), and principles-based approaches for calculating statutory reserves for all life insurers. CARVM applied to fixed annuities and VA guaranteed minimum death benefits was in effect much earlier under Actuarial Guidelines XXXIII (AG-33) and XXXIV (AG-34). Interested readers are referred to Sharp (1999b,a) for details.
2. Risk-based capital requirement (RBC)
   The NAIC's RBC regime was established in the 1990s as an early warning system for US insurance regulators. The RBC requirement specifies the minimum amount of capital an insurer is required to hold in order to support its overall business operation in consideration of its size and risk profile. If an insurer

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does not meet the RBC requirement, then regulators have the legal authority to take preventive and corrective measures in order to protect policyholders and the stability of the insurance market. The NAIC developed a system of RBC formulas based on three major areas: (1) Asset Risk; (2) Underwriting Risk; and (3) Other Risk, which are called C-1, C-2, C-3 risks by practitioners. The C-3 category includes interest rate and market risks. Due to the complexity of varying exposure of different product designs and funding strategies, the NAIC implemented the RBC requirements for C-3 risk in two phases. Phase I addressed interest rate risk for single premium life insurance and annuities including deferred and immediate annuities, guaranteed investment certificates, etc. Since 2003, the NAIC has adopted several recommendations and revisions from the AAA for Phase II capital standards for variable annuities and other products with equity-related risks.

There is extensive literature on no-arbitrage pricing of various types of variable annuity guaranteed benefits. To name a few, a model for the guaranteed minimum death benefit (GMDB) was first introduced by Milevsky and Posner (2001) and later extended in consideration of rollup and ratchet options in Ulm (2008) and Ulm (2010). The valuation of guaranteed minimum withdrawal benefits was considered in Milevsky and Salisbury (2006) from a policyholder's perspective. Feng and Volkmer (in press) extended their work to consider the valuation from an insurer's perspective. A recent work by Huang et al. (2014) investigated optimal initiation of withdrawals for a guaranteed lifetime withdrawal benefit. The valuation of guaranteed minimum income benefits was introduced in Marshall et al. (2010). Ng and Li (2011) developed risk-neutral pricing of guaranteed benefits under regime-switching models. Bernard et al. (2014) used American option techniques to derive the optimal policyholder surrender strategy based on risk-neutral prices. Bauer et al. (2008), Bacinello et al. (2011), Pitacco (in press) and Ballotta and Haberman (2006) provided general frameworks under which various guaranteed benefits can be evaluated. However, it should be pointed out that pricing actuaries rarely use no-arbitrage pricing theory to determine fees and charges in practice. Instead, they run stochastic projections of cash flows to determine pathwise pricing metrics such as internal rate of return on capital, net present value of profits, etc. Nevertheless, no-arbitrage pricing in the literature can provide potential solutions to financial reporting with hedging models, for which the nested simulation is a major technical concern in the industry.

The main focus of the existing academic literature has been to address the fundamental question of how much should policyholders be charged for the guaranteed benefits. Very little attention has been paid to an equally important question, which is how much reserve and capital an insurer should hold to cover expected and unexpected losses. Hardy (2003) was among the first work in the literature to address such a question, and systematically exploited risk management of equity-linked insurance. As the North American insurance industry has undertaken rapid development of financial reporting standards in the past decade, there has been a wide gap in the actuarial literature with regard to quantitative models of financial reporting. Recently, Bauer et al. (2012) proposed various Monte Carlo methods to determine the European solvency capital requirements.

In this paper, we investigate quantitative models for insurer's liabilities from variable annuity guaranteed benefits. However, readers should bear in mind that actual statutory reserve and capital calculations mandate detailed accounting standards, which can be complex and tedious with various product lines. As the purpose of this paper is to explore the quantitative structure of financial reporting, we shall take a minimalist approach and focus only on a few essential elements that involve

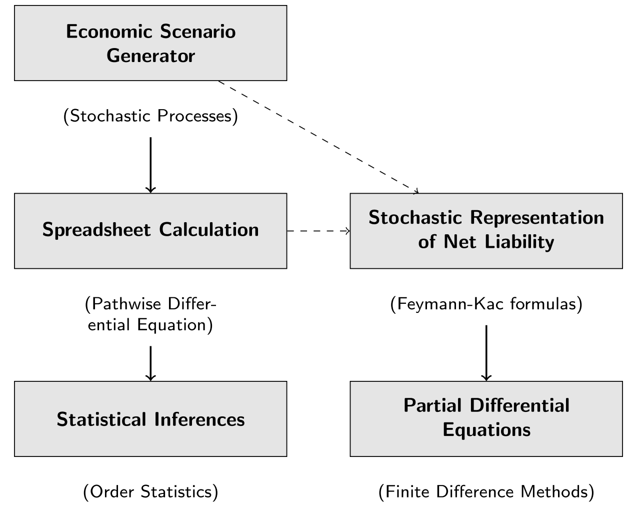

Fig. 1. Two computational methods of risk measures.

stochastic components. There are significant differences between the calculation of C-3 Phase II risk-based capital requirement (Total Asset Requirement) and that of the AG-43 reserve (Conditional Tail Expectation amount) from the viewpoint of practitioners.[1] Reserving is often done on a seriatim (contract by contract) basis whereas the RBC calculation is performed at the aggregate level. However, from a mathematical point of view, both share very similar quantitative structure and dependency on their underlying stochastic processes, which can be summarized in three steps as shown in the left column in Fig. 1. In this paper, we only use the AG-43 reserve as a model example of financial reporting.

Step 1: Use either pre-packaged scenarios or internally built stochastic models (called economic scenario generators) for all risk factors driving the insurer's asset and liability portfolio. These stochastic models have to be calibrated to meet regulatory standards. Generate a variety of sample paths of the stochastic models over a projection period.

Step 2: Use spreadsheets or third-party vendor software to determine account values for an individual or aggregate contract. Under each scenario of account values, follow certain accounting standards to determine the accumulated profit/deficiency for the entire projection length.

Step 3: Repeat Step 2 for each scenario many times to generate an empirical distribution of accumulated surplus/deficiency or other performance metrics. Apply order statistics to estimate certain risk metrics, such as quantile/conditional tail expectation, which form the basis of reserve or capital requirement.

The first goal of this paper is to formulate the aforementioned standard practice in a quantitative model. Even though the reserving method (CARVM) has become an industrial standard over the past decade, there is no existing research on the mathematical structure and interpretation of such a practice. As spreadsheet calculations in essence are based on pathwise defined recursive relations, we can integrate this information with the underlying stochastic models to determine a stochastic representation of an insurer' accumulated profit/deficiency.

[1] For example, the calculation required by AG 43 is performed on a pre-tax basis whereas that required by C-3 Phase II is done on an after-tax basis, treatment of standard scenarios, the RBC calculation is based on 90% CTE whereas the AG-43 is based on 70% CTE, etc. Detailed discussions of the similarities and differences between the two requirements can be found in Life Practice Note Steering Committee (2009).