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Robust optimal portfolio and proportional reinsurance for an insurer under a CEV model[☆]

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HIGHLIGHTS

- A robust optimal portfolio and reinsurance problem under a constant elasticity of variance (CEV) model is investigated.
- The closed-form expressions for the optimal strategies and the corresponding value functions are obtained, and the verification theorem is strictly proved.
- The classic Cramér–Lundberg risk process and its diffusion approximation are both discussed.
- The impact of uncertainty about the diffusion risk and the jump risk are considered simultaneously.
- The DPP approach and PDE technique are used.

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ABSTRACT

We investigate a robust optimal portfolio and reinsurance problem under a Cramér–Lundberg risk model for an ambiguity-averse insurer (AAI), who worries about uncertainty in model parameters. Assume that the AAI is allowed to purchase proportional reinsurance and invest his (or her) surplus in a financial market consisting of one risk-free asset and one risky asset whose price is modeled by a constant elasticity of variance (CEV) model. Using techniques of stochastic control, we first derive the closed-form expressions of the optimal strategies and the corresponding value functions for exponential utility function both in the classic compound Poisson risk process and its diffusion approximation, and then the verification theorem is given. Finally, we present numerical examples to illustrate the effects of model parameters on the optimal investment and reinsurance strategies.

1. Introduction

Insurance companies have been major institutions in worldwide financial markets. They are actively involved in trading activities in various financial markets. Consequently, the optimal asset allocation problem is important for insurance companies. The key difference between an optimal asset allocation problem for insurance companies and its financial counterparts is the presence of insurance liabilities, which are mainly due to insurance claims. In the business world with fierce competition, insurance companies are trying to take various strategies to increase their reserve and minimize their risk. In general, insurance companies face two sources of risks: the risk arising from unexpectedly large insurance claims and the market risk arising from risky investments in financial markets. Reinsurance is one of the ways that insurance companies effectively transfer parts of their risks due to insurance claims. To reduce the market risk, the investor often invests some risk-free assets such as short-term bonds and money market funds. Therefore, the optimal portfolio and reinsurance problems with various objectives in insurance risk management have attracted a lot of attention in the past few years, and a significant amount of works have been done concerning this topic. Some scholars focused on maximizing the expected utility of insurance companies' terminal wealth. See, for example, Browne (1995), Yang and Zhang (2005), Bai and Guo (2008), and Liang et al. (2011) etc.

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During the past decades, some scholars have taken model uncertainty into account when dealing with optimal portfolio selection problem. In reality, no consensus has been reached on which model should be used when investor tries to obtain the optimal portfolio. In addition, it is inevitable to make errors in such model. Hence, model uncertainty is an important issue in any modeling exercises. The idea behind dealing with model uncertainty is game theory. Many scholars focused on these works. Mataramvura and Øksendal (2008) studied the problem of looking for risk minimized portfolio in a jump–diffusion market; Zhang and Siu (2009) investigated the optimal investment and reinsurance problems of an insurer; a similar article is Lin et al. (2012), in which a risk-free asset was included into their model. A general feature of model uncertainty is characterized by a family of probability measures, or scenarios. This leads to the study of robust models, where one seeks an optimal strategy among a family of possible scenarios. The rationale of this topic is to take ambiguity aversion into consideration. The concept of ambiguity aversion can be traced back to Anderson et al. (1999). Uppal and Wang (2003) extended the results of Anderson et al. (1999) under robustness framework with different levels of ambiguity. For other articles in this field we refer readers to Maenhout (2004, 2006) and Liu (2010).

Among most of the articles mentioned above, the volatilities of risky assets' prices are assumed to be a constant or deterministic function. However, the risky assets' prices may have different features in the real world and many empirical studies (see e.g., Beckers, 1980, Campbell, 1987, Hobson and Rogers, 1998, and the references therein) have shown that empirical evidences tend to support the stochastic volatility model for the risky assets. Heston (1993) used a Cox–Ingersoll–Ross (CIR) process to characterize the volatility of the risky asset. Since then, the Heston's model has been adopted in various literature to investigate the optimal investment and/or consumption problems for investors, see Liu and Pan (2003), Chacko and Viceira (2005), Kraft (2005) and Liu (2007). In recent years, the Heston's model was also widely used in the field of insurance. Zhao et al. (2013) studied the optimal reinsurance and investment problem for an insurer under Heston model, and the objective of the insurer is to maximize the expected exponential utility of terminal wealth. Yi et al. (2013) investigated the robust optimal reinsurance and investment strategies for an insurer with individual preferences when facing model uncertainty.

Apart from the Heston's model, the CEV model is also a good tool to describe the stock price. Darius (2005) first studied the optimal portfolio selection problem under the CEV model. Xiao et al. (2007) and Gao (2009) studied pension investment problems with CEV risky asset price processes. Gu et al. (2010), Lin and Li (2011) and Liang et al. (2012) investigated the optimal reinsurance and investment strategies under CEV models. However, none of the works above proved the verification theorem. In Gu et al. (2012), the authors solved the optimal reinsurance and investment problem under a CEV model with diffusion approximation risk process, and provided the verification theorem. Inspired by these works mentioned above, in our article, we dedicate to obtaining the robust optimal reinsurance and investment strategies for an insurer under a CEV model, and shall prove the verification theorem under robust framework which is different from the case in Gu et al. (2012).

To the best of our knowledge, there are no published works considering the effect of uncertainty about the diffusion risk arising from risky investments in financial markets and the jump risk arising from unexpectedly large insurance claims simultaneously. In our article, we aim to consider the optimization problem which includes two types of uncertainties. We begin with a general utility function, then followed by a verification theorem. Furthermore, as of the exponential utility function, the sufficient conditions were offered under which the closed-form solution for optimal strategies is obtained. Summarizing and comparing with the existing literature, we conclude that there are two main contributions of this article:

(i) A robust optimal reinsurance and investment problem under a CEV model with exponential utility is considered, and the verification result for this model is provided and has significant differences from the result in Gu et al. (2012).
(ii) The impact of model uncertainty of the jump risk arising from unexpectedly large insurance claims is investigated, which Yi et al. (2013) did not consider.

The rest of this article is organized as follows. Section 2 presents the assumptions and the model dynamics. In Section 3, a robust control problem for an AAI with exponential utility is presented, and derives the closed-form expressions for the optimal strategies and the corresponding value function with a technical condition. In Section 4, we derive a closed-form solution to the robust control problem in a diffusion approximation risk process. Section 5 analyzes our results using numerical illustration. Section 6 gives more thoughts and our future work. In this section, we analyze the feasibility of measuring risk preference by other utility functions for an insurer. Then we discuss how to handle the general case by using numerical method.

2. The financial market model and assumptions

In this section, we will give models and some basic assumptions. We suppose that in this financial market all assets can be traded continuously over time, no transaction costs or taxes are involved in trading. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\in[0,T]}$. T is a positive finite constant representing the terminal time. Any decision made at time t is based on $\mathcal{F}_t$ which can be interpreted as the information available until time t. Thus $T-t$ can be understood as the horizon at time t.

Without loss of generality, we assume that there are two assets available for an insurer in the financial market: a bond and a stock. The price of the bond is described by

$$dB(t) = rB(t)dt, \tag{2.1}$$

here $r > 0$ is the risk-free interest rate. The price process of the stock is given by the CEV model

$$dS(t) = S(t)\big[\mu dt + \sigma S^{\beta}(t)dW(t)\big], \tag{2.2}$$

where $\mu(> r)$ is the expected instantaneous return rate of the stock; σ is a positive constant; β is called the elasticity parameter; $\sigma S^{\beta}(t)$ is the instantaneous volatility; $W(t)$ is a standard Brownian motion under measure P. As in Xiao et al. (2007), Geman and Shih (2009), Dias and Nunes (2011) and Gu et al. (2012), we may assume that $\beta \geq 0$.

We suppose that the risk process of the insurer is described by the Cramér–Lundberg model:

$$dR(t) := cdt - d\sum_{i=1}^{N(t)} Y_i = cdt - \int_0^{\infty} yN(dt, dy), \tag{2.3}$$

where c is the premium rate, $C(t) := \sum_{i=1}^{N(t)} Y_i$ represents the aggregate claims up to time t, $\{N(t), t \geq 0\}$ is a homogeneous Poisson process with intensity λ, and $\{Y_i, i \geq 1\}$ is a sequence of positive independent and identically distributed random variables with common distribution $F(y)$ and finite first and second moments m_1 and m_2 respectively, $N(dt, dy)$ is the Poisson random measure corresponding to the compound Poisson process $C(t)$. Assume that the premium rate c is calculated according to the expected value principle, that is, $c = (1+\eta_1)\lambda m_1$, where $\eta_1 > 0$ is the relative safety loading of the insurer. In addition, we assume