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Semi-static hedging of variable annuities

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ABSTRACT

This paper focuses on hedging financial risk in variable annuities with guarantees. We show that insurers should incorporate the specificity of the periodic payment of variable annuities fees to best hedge embedded guarantees and should focus on hedging the net liability. We develop a new hedging strategy based on semi-static hedging techniques, which takes into account the periodically collected fees, and confirm that it is more effective than delta-hedging with same rebalancing dates, as well as traditional semi-static hedging strategies that do not consider the specificity of the payments of fees in their optimization. It is also verified that short-selling or using put options as hedging instruments allows more effective hedging.

1. Introduction

Life expectancies are steadily increasing and the population faces the need to guarantee a sustainable income after retirement. It leads to a natural demand for variable annuities (VAs) that offer guaranteed income for the post-retirement life. The first VAs were sold in the U.S. in 1952, then introduced in Japan in the 1990s, in Europe in 2000 (Hardy, 2003) and in China in 2011. In the U.S., although the sales of VAs have slightly decreased between 2011 and 2014, they still represent about 60% of the total sales of annuities in 2014.[1] In Japan, sales of new VAs have increased until 2006 and then decreased since then. In Europe, VAs began to be marketed only since the mid 2000s. At the end of 2009 there were 168 billion euros of technical provisions relating to VAs, and 188 billion at the end of 2010 (EIOPA, 2011). They are most prominent in the UK with 1.4 billion pounds in VA sales in 2012 (Winkler, 2013) and an overall increasing sales trend. In China, the VA sales' volume is still very small. A recent state-of-the-art on the VAs market can be found in Haefeli (2013) with figures and charts illustrating the VAs market in the U.S., Canada, Europe, and Japan between 1990 and 2011.

A typical VA contract consists of an accumulation phase, during which the policyholder invests an initial premium and possibly subsequent premiums into a basket of invested funds. At the end of the accumulation phase, the policyholder may receive a lump sum or may annuitize it to provide retirement income. VAs contain both investment and insurance features. They improve upon traditional fixed annuity contracts that offer a stream of fixed retirement income, as policyholders of VAs could expect higher returns in a bull market. VAs also provide some protection against downside risk with several kinds of additional benefit riders, which are sold with the VA. Traditional annuities could simply be hedged by an appropriate portfolio of bonds. But the presence of protection benefits requires more sophisticated hedging strategies.

The simplest benefit is the Guaranteed Minimum Accumulation Benefit (GMAB). It guarantees a lump sum on a specific future date or anniversary. But it is not as popular as the following more complex benefits; Guaranteed Minimum Income Benefits

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[1] Specifically, according to LIMRA Secure Retirement Institute (www.limra.com), VA sales in the U.S. increased from $137 billion in 2005 to $184 billion in 2007 (prior to the financial crisis), then decreased to a minimum of $128 billion in 2009. In the last four years, sales of annuities have increased from $220 billion in 2012 to $236 billion in 2014. During that period, VA sales tend to slightly decrease from $147 billion to $140 billion, whereas fixed annuities sales increased from $72 billion in 2012 to $96 billion in 2014. But total net assets from VAs are close to two trillion U.S. dollars (Source: Insured Retirement Institute http://irionline.org/newsroom/newsroom-detail-view/iri-issues-fourth-quarter-and-year-end-2014-annuity-sales-report).

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(GMIB) and Guaranteed Minimum Withdrawal Benefits (GMWB). GMIB guarantee a stream of lifetime annuity income after the policyholder's annuitization decision is made. GMWB guarantee the ability to withdraw a specified percentage of the benefit base during a specified number of years, or it could be a lifetime benefit (Kling et al., 2011). A Guaranteed Minimum Death Benefit (GMDB) guarantees a specified lump sum benefit at the time of policyholder death. See Hardy (2003) for more details.

There are lots of studies on the pricing of VAs and how to find the fair fee; Milevsky and Posner (2001) and Bacinello (2003) investigate the valuation of GMDB in VAs using risk-neutral pricing. Lin et al. (2009) price simple guarantees in VAs in a regime switching model. Marshall et al. (2010) study the value of a GMIB in a complete market, and the sensitivities of the value of GMIB to the financial variables are examined. They suggest that the fee rate charged by insurance companies for GMIB may be too low. GMWB are studied intensively by several authors including Milevsky and Salisbury (2006), Chen et al. (2008), Dai et al. (2008), Liu (2010), and Kolkiewicz and Liu (2012). Milevsky and Salisbury (2006) show that GMWB fees charged in the market are too low and not sustainable. They argue that the fees have to be increased or the product design should be changed to avoid arbitrage. Chen et al. (2008) also conclude that normally charged GMWB fees are not enough to cover the cost of hedging. Bauer et al. (2008) and Bacinello et al. (2011) propose a general valuation framework for VAs.

There are fewer papers on hedging, although hedging embedded guarantees of VAs are a challenging and crucial problem for insurers. There are three main sources of risk in VAs: mortality risk, policyholder behavioral risk, and financial risk. Here, we discuss these three risk sources, then focus on financial risk in the rest of the paper. When mortality risk is fully diversifiable, it is straightforward to hedge mortality risk by selling independent policies to a group of policyholders with similar risk of death. However, mortality risk cannot always be diversified and VAs are exposed to longevity risk (i.e., a systematic change in mortality risk affecting all of the population simultaneously). It is a topic of research by itself and a thorough analysis of longevity risk in VAs can be found in the studies such as Ngai and Sherris (2011), Hanewald et al. (2012), Gatzert and Wesker (2012), and Fung et al. (2013).

Behavioral risk faced by the insurer in VAs arises from the uncertainty of policyholder's decisions such as choice of surrender, partial withdrawal, annuitization, reallocation, additional contributions, and so on. In general, behavioral risk is difficult to hedge. Under the assumption that investors do not act optimally and base their decisions on non-financial variables, behavioral risk can be diversified similarly as mortality risk by using a deterministic decision making process using historical statistics, which state, for instance, that x% of the policyholders follow a given behavior in a specific situation. However, there is an empirical evidence that policyholders may act optimally, or at least that their decisions are correlated with some market factors and depend on the moneyness of the guarantee, so that all behavioral risks may not be fully diversified (see the empirical study of Knoller et al., forthcoming). Kling et al. (2014) study the impact of behavioral risk on the pricing, as well as on the hedging effectiveness of VAs. They consider various assumptions on behaviors; from deterministic to optimal decision making. Interestingly, the impact of model misspecification on policyholders' behaviors depends highly on the design of the policy. For example, the effect of the surrender decision is more important in VAs without ratchets. MacKay et al. (forthcoming) design VAs that are never optimal to surrender. See also Augustyniak (2013) for a study of the effect of lapsations on the hedging effectiveness of the Guaranteed Minimum Maturity Benefit (GMMB).

In this paper, mortality risk, longevity risk, and policyholder behavioral risk are ignored, and we focus on hedging financial risk. It can be done via delta-hedging, semi-static hedging, or static hedging. Guarantees in VAs are similar to options (financial derivatives) on the fund value and the insurer plays the role of option writer. However, they are also very different from standard derivatives; a crucial difference is that the costs of these options are not paid upfront like initial option premiums. By contrast, fees are paid periodically as a percentage of the fund value throughout the life of the contract. After finding the suitable level of fees to cover the guarantees by fair pricing of VAs, hedging the guarantees is how to utilize the collected fees to match the payoff of the guarantees when they should be paid. The main difficulty is that the option premiums for the guarantees are not paid upfront so that the uncertainty and risk are inherent in the option premiums, as well as the payoff. Therefore, both components should be hedged. In addition, there is an unavoidable mismatch between the (random) value of fees collected from the policyholder's account and the hedging cost, because the value of collected fees and the cost of hedging move in opposite directions. When the policyholder's account value is high, the value of the fee is also high while the value of the embedded option in the guarantee is low. If the account value is low, the value of the fee is low but the insurer needs more money for the guaranteed benefits because the value of embedded option is high. This mismatch represents a real challenge for hedging guarantees in VAs. It is also exactly the reason why hedging guarantees in VAs differ from hedging standard options (for which fees are paid at inception only).

The most common approach to hedge financial risk is to perform a dynamic delta-hedging (Kling et al., 2014) and (Kling et al., 2011) to replicate the embedded guarantees. When hedging a guarantee that depends on a tradable fund F_t (or at least on a replicable fund F_t), the delta hedging portfolio consists of a number of shares of fund equal to the delta of the value of guarantee (i.e., the sensitivity of the value of the guarantee to a change in the underlying price). If it is assumed that the market is complete, with no transaction costs and the rebalancing can be done continuously over time, then, a dynamic delta-hedging strategy achieves a perfect hedge of the guarantees at the time they must be paid out. An alternative popular dynamic hedging is based on mean variance criteria and is sometimes referred as "optimal dynamic hedging" (Papageorgiou et al., 2008; Hocquard et al., 2015, 2012). This latter technique can be applied in incomplete markets.

Instead of dynamic hedging, Hardy (2003, 2000) and Marshall et al. (2010, 2012) investigate static hedging and suggest replicating maturity guarantees with a static position in put options. Static hedging consists of taking positions at inception in a portfolio of financial instruments that are traded in the market (at least over-the-counter) so that the future cash-flows of the VA match the future cash-flows of the hedge as well as possible. Static hedging strategies have no intermediary costs between the inception and the maturity of the benefits, and tend to be highly robust to model risk because no rebalancing is involved. However, there are several issues with this approach, in particular the non-liquidity (and non-availability) of the long-term options that are needed to match the long-term guarantees as well as their exposure to counterparty risk. Also, if the guarantees are path-dependent, it is hard to hedge with static hedging using the European path-independent options available in the market. Finally, static hedging of VAs tends to forget about the specificity of the options embedded in VAs. Their option premiums (fees) are paid in a periodic way affecting the fund value. Thus a more natural hedging strategy should account for the periodicity of these fees. Moreover, in a static strategy, the insurer must borrow a large amount of money at the inception of the contract to construct the hedging portfolio. This borrowed money will then potentially be offset by the future fees collected as a percentage of the fund. But the insurer is subject to the risk that the fees collected in the