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Addendum to 'The multi-year non-life insurance risk in the additive reserving model' [Insurance Math. Econom. 52(3) (2013) 590–598]: Quantification of multi-year non-life insurance risk in chain ladder reserving models

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HIGHLIGHTS

- Closed-form analytical expressions for multi-year non-life insurance risk.
- Approach based on the chain ladder method using a first-order Taylor approximation.
- Previous accident years and new accident years in one integrated analytical model.
- Deduction of well-known results for one-year and ultimate reserve risk from our model.
- Case study demonstrating the applicability and usefulness of our results.

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ABSTRACT

This is the first study to derive closed-form analytical expressions for multi-year non-life insurance risk in the chain ladder model. Extending on previous research on the additive reserving model, we define multi-year risk via prediction errors of multi-year claims development results including both observed and future accident years. A resampling argument and a first-order Taylor approximation address the quantification of estimation errors and multiplicative dependencies in the chain ladder framework, respectively. From our generalized multi-year approach, we deduce estimators for reserve and premium risks in multi-year view and their implicit correlation. We reproduce well-known results from literature for the special cases of one-year and ultimo view. Further, we comment on how to obtain estimators for generalized versions of the chain ladder method. A case study demonstrates the applicability of our analytical formulae.

1. Introduction

Internal risk assessment of multiple years is a key feature of strategic management decisions and ORSA reporting under Solvency II for non-life insurance companies (see, e.g., Diers et al., 2013). In general, multi-year non-life insurance risk consists of a reserve risk component, measuring the uncertainty in the settlement of outstanding claims from previous accident years, and a premium risk component for the uncertainty in future claims (for the non-life insurance risk in a one-year view see, e.g., Ohlsson and Lauzeningks, 2009). A valid risk quantification approach should embed both types of risks to account for their inherent dependencies. In this paper, we define this overall uncertainty in the future non-life business by the variation of the multi-year future claims development results in the underlying claims reserving method. In this context, Diers and Linde (2013) provide analytically closed formulae for the non-life insurance risk subject to an arbitrary number of years under the additive loss reserving model.

For the chain ladder framework, Böhm and Glaab (2006) and Merz and Wüthrich (2008) present analytical approaches for calculating the variability of future claims development results.

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However, Merz and Wüthrich (2008) neglect premium risk and restrict results to the two special cases of one-year and ultimo view. Although Böhm and Glaab (2006) do consider volume measures for the next calendar year, they neither explicitly model the premium risk nor allow for any multi-year time horizon. To our knowledge, no closed-form analytical expressions are yet available to calculate the variability of a general multi-year claims development result for the chain ladder method. It is worth mentioning that simulation techniques exist to obtain estimates for the multi-year non-life insurance risk, see Diers et al. (2013) for instance.

The aim of this paper is to provide for the first time such analytical closed-form expressions for multi-year non-life insurance risk in this line of research. We derive our main results from a slight extension of the classical chain ladder model, allowing previous and new accident years to be combined in one integrated approach. The multiplicative chain ladder methodology makes variance terms for the claims development result intractable. Because of this, we approximate them by the well-established technique of first-order Taylor expansions and inherently account for the non-trivial estimation variance component through a conditional resampling argument. We demonstrate that our flexible multi-year approach reproduces the well-known results for reserve risk in one-year and ultimo view. Furthermore, we deduce formulae for the variety of interesting cases, as in Diers and Linde (2013), such as reserve and premium risks in multi-year view and their inherent correlation. We comment on how to establish analogous results for generalized versions of the chain ladder method that incorporate smoothing of chain ladder factors and exclusion of selected development factors from the estimation process. The analytical findings can support multi-year internal risk models within strategic management and decision making as well as the ORSA process of Solvency II. A case study underlines the applicability of the formulae.

This paper is organized as follows. After defining the multi-year non-life insurance risk in the extended chain ladder reserving model in Section 2, we derive the main results for the multi-year view in Section 3. Section 4 describes the applicability of the techniques when using generalized versions of the chain ladder method. In Section 5, we illustrate the theoretical results of the previous chapters by means of a numerical example. Section 6 concludes the paper.

2. Multi-year risk in the chain ladder model

The chain ladder model by Mack (1993) is probably the claims reserving model most widely used among practitioners. Based on the idea of Böhm and Glaab (2006), we extend the original chain ladder model as in Diers et al. (2013) to allow for consideration of future accident years as well. By analogy to Diers and Linde (2013), in this chapter we derive the m-year claims development result and its prediction uncertainty under this framework. Throughout this work, we choose notation and definitions to be consistent with Diers and Linde (2013).

2.1. Model framework

Let $n, m \in \mathbb{N}$ denote the numbers of observed and future accident years, respectively, and $C_{i,k} > 0$ the cumulative payments for a single accident year $i \in \{1, \dots, n+m\}$ up to a development year $k \in \{1, \dots, n\}$. Moreover, for all accident years $i = 1, \dots, n+m$, let $v_i =: C_{i,0} > 0$ be suitable (deterministic) volume measures (e.g. premiums or number of contracts) that are assumed known even for future accident years. At the end of period $T = n$, the available data then become $\mathscr{D}_n := \mathscr{V} \cup \Delta_n$ with the full set of volume measures $\mathscr{V} := \{C_{i,0}, 1 \le i \le n+m\}$ and the so far observed claims triangle $\Delta_n := \{C_{i,k}, 1 \le i \le n, 1 \le k \le n-i+1\}$. For any time horizon $t \in \{0, \dots, m\}$, we further introduce

$$
\begin{aligned}
{}^{(n+t)}\mathscr{A}_i &:= \left\{C_{i,0}, \dots, C_{i,\min\{n-i+1+t,n\}}\right\}, \\
&\quad i \in \{1, \dots, n+t\}, \\
{}^{(n+t)}\mathscr{B}_k &:= \left\{C_{i,j}\colon 1 \le i \le n+t+1, 0 \le j \le k, i+j \le n+t+1\right\}, \\
&\quad k \in \{0, \dots, n\}.
\end{aligned}
$$

The set ${}^{(n+t)}\mathscr{A}_i$ contains all observed information at time $T = n+t$ for a single accident year i. Similarly, ${}^{(n+t)}\mathscr{B}_k$ is the then observed information on all accident years up to development year k.

We extend the original chain ladder model by Mack (1993) with a volume model for the claims payments in the first development year, keeping it distribution-free.

Definition 2.1 (*Extended Chain Ladder Model*)**.** The following assumptions define the *extended chain ladder model*:

(CL1) Cumulative payments $C_{i,k}$ in different accident years $i \in \{1, \dots, n+m\}$ are independent.
(CL2) For each $k \in \{1, \dots, n\}$, there exists f_k such that $\mathbb{E}\left[C_{i,k} \mid C_{i,0}, \dots, C_{i,k-1}\right] = f_k C_{i,k-1}$ for all $i = 1, \dots, n+m$.
(CL3) For each $k \in \{1, \dots, n\}$, there exists $\sigma_k^2 > 0$ such that $\mathbb{V}\left[C_{i,k} \mid C_{i,0}, \dots, C_{i,k-1}\right] = \sigma_k^2 C_{i,k-1}$ for all $i = 1, \dots, n+m$.

The choice $m = 0$ yields the original chain ladder model according to Mack (1993) with volume measures. It is common to rewrite assumptions (CL2) and (CL3) in terms of the single development factors $F_{i,k} := C_{i,k}/C_{i,k-1}$ for $k = 1, \dots, n$:

$$
\mathbb{E}\left[F_{i,k} \mid C_{i,0}, \dots, C_{i,k-1}\right] = f_k,
$$

$$
\mathbb{V}\left[F_{i,k} \mid C_{i,0}, \dots, C_{i,k-1}\right] = \frac{\sigma_k^2}{C_{i,k-1}}.
$$

In addition to the original chain ladder framework, the parameters $F_{i,1}$ represent the ratio of first-year incremental payments and the volume measures for each accident year i. Apart from interpretation, they exhibit the same mathematical properties.

We naturally expand the classical estimators (see Mack, 2009) to the first development year $k = 1$, i.e. we estimate model parameters f_k, $k = 1, \dots, n$, and σ_k^2, $k = 1, \dots, n-1$, based on $\mathscr{D}_n$ by

$$
{}^{(n)}\hat{f}_k := \frac{\sum_{i=1}^{n-k+1} C_{i,k}}{\sum_{i=1}^{n-k+1} C_{i,k-1}},
$$

$$
\widehat{\sigma}_k^2 := \frac{1}{n-k} \sum_{i=1}^{n-k+1} C_{i,k-1} \left(\frac{C_{i,k}}{C_{i,k-1}} - {}^{(n)}\hat{f}_k \right)^2.
$$

As in the original model, we call ${}^{(n)}\hat{f}_k$ *chain ladder factors*. Different possibilities can be chosen for $\widehat{\sigma}_n^2$ (see, e.g., Mack, 2002 and DAV, 2008). For the case study in Section 5, we choose $\widehat{\sigma}_n^2 := \min\left\{\widehat{\sigma}_{n-3}^2, \widehat{\sigma}_{n-2}^2, \widehat{\sigma}_{n-1}^2\right\}$.

Obviously, the well-known properties of the development factors and their classical estimators apply to the extended model (see Mack, 2002).

Lemma 2.2 (*Properties in the Extended Chain Ladder Model*)**.** *The following properties hold:*

(i) $\mathbb{E}\left[{}^{(n)}\hat{f}_k \,\middle|\, {}^{(n)}\mathscr{B}_{k-1}\right] = f_k$ *for all* $k = 1, \dots, n$, *i.e. the chain ladder factors are conditionally unbiased estimators for the parameters* f_k *given* ${}^{(n)}\mathscr{B}_{k-1}$.