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Hedging of unit-linked life insurance contracts with unobservable mortality hazard rate via local risk-minimization

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ABSTRACT

In this paper we investigate the local risk-minimization approach for a combined financial-insurance model where there are restrictions on the information available to the insurance company. In particular we assume that, at any time, the insurance company may observe the number of deaths from a specific portfolio of insured individuals but not the mortality hazard rate. We consider a financial market driven by a general semimartingale and we aim to hedge unit-linked life insurance contracts via the local risk-minimization approach under partial information. The Föllmer–Schweizer decomposition of the insurance claim and explicit formulas for the optimal strategy for *pure endowment* and *term insurance* contracts are provided in terms of the projection of the survival process on the information flow. Moreover, in a Markovian framework, this leads to a filtering problem with point process observations.

1. Introduction

The paper addresses the problem of computing locally risk-minimizing hedging strategies for unit-linked life insurance contracts under partial information. In these contracts, insurance benefits depend on the price of a specific risky asset and payments are made according to the occurrence of some events related to the stochastic life-length of the policy-holder. In particular we consider a portfolio of l_a insured individuals, having all the same age a. Hence, insurance contracts can be considered as contingent claims in an incomplete combined financial-insurance market model, defined on the product of two independent filtered probability spaces: the first one, denoted by $(\Omega_1, \mathcal{F}, \mathbf{P}_1)$ endowed with a filtration $\mathbb{F} := \{\mathcal{F}_t,\ t \geq 0\}$, is used to model the financial market, while the second one, $(\Omega_2, \mathcal{G}, \mathbf{P}_2)$ endowed with a filtration $\mathbb{G} := \{\mathcal{G}_t,\ t \geq 0\}$, describes the insurance portfolio.

In general, incompleteness occurs when the number of assets traded on the market is lower than that of random sources, see e.g. Bjork (2004, Chapter 8). This is, for instance, the case of insurance claims which are linked to both financial markets and other sources of randomness that are stochastically independent of the financial markets.

Since we consider an insurance market model which is independent of the underlying financial market, we apply the (local) risk-minimization approach for deriving hedging strategies that reduce the risk. This is a quadratic hedging method which keeps the replication constraint and looks for hedging strategies (in general not self-financing) with minimal cost.

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The concept of risk-minimizing hedging strategies was introduced in Föllmer and Sondermann (1986) in the financial framework only. At the beginning it was formulated assuming that the historical probability measure was a martingale measure. In this context, some results were obtained in the case of full information by Föllmer and Sondermann (1986) and Schweizer (2001), and under partial information by Schweizer (1994b) using projection techniques, and more recently in Ceci et al. (2014c) where the authors provide a suitable Galtchouk–Kunita–Watanabe decomposition of the contingent claim that works in a partial information framework. When the historical probability measure does not furnish the martingale measure, i.e. the asset prices dynamics are semimartingales, the theory of risk minimization does not hold and a weaker formulation, namely *local risk-minimization*, is required (see e.g. Föllmer and Schweizer, 1991 and Schweizer, 2001).

Concerning the local risk-minimization approach under partial information, there are less results in the literature, as far as we are aware. In particular, we mention: Föllmer and Schweizer (1991), where the optimal strategy is obtained via predictable projections and enlargements of filtrations that make the financial market complete; Ceci et al. (2014b), which provides a suitable Föllmer–Schweizer decomposition of the contingent claim; Ceci et al. (2014), where the authors derive a full description of the optimal strategy under some conditions on the filtrations; an application to the case of defaultable markets in the sense of Föllmer and Schweizer (1991) can be found in Biagini and Cretarola (2009). Finally, we also quote Menoukeu Pamen and Momeya (2011) for local risk-minimization applied to Markov modulate exponential Lévy models under complete and partial information.

The (local) risk-minimization approach has also been investigated under the so-called *benchmark approach*, a modeling framework that employs the numéraire portfolio as reference unit, instead of the riskless asset. More precisely, in Biagini et al. (2014) the authors consider the full information setting, the restricted information case is studied in Ceci et al. (2014a), and finally in Du and Platen (2014) the authors propose a different formulation of the problem for contingent claims which are not square-integrable.

The theory of (local) risk-minimization has been recently extended to the insurance framework, where the market is affected by both mortality and catastrophic risks. In Møller (1998) and Vandaele and Vanmaele (2008) the authors study the hedging problem of unit-linked life insurance contracts in the Black & Scholes and in the Lévy financial market model respectively, under full information on both the insurance market and the financial one. In particular in Vandaele and Vanmaele (2008) the authors discuss the same model analyzed in Riesner (2006). Moreover, in Ceci et al. (2014a) the problem is solved for a general semimartingale financial model under partial information on the financial market using the benchmark approach. In all these papers lifetimes of the l_a insured individuals are represented by i.i.d. nonnegative random variables with known hazard rate.

The novelty of this paper consists in considering a combined financial-insurance model where there are restrictions on the information concerning the insurance market. As a matter of fact, we assume that, at time t, the insurance company may observe the total number of deaths N_t occurred until t but not the mortality hazard rate, which depends on an unknown stochastic factor X. More precisely, we assume that lifetimes of each individual are conditionally independent, given the whole filtration generated by X, $\mathcal{G}^X_\infty := \sigma\{X_u,\ u \geq 0\}$, and with the same hazard rate process $\lambda_a(t, X_t)$. Denoting by $\mathbb{G}^N$ the filtration generated by the process N which counts the number of deaths, then on the combined financial-insurance market the information flow available to the insurance company is formally described by the filtration $\widetilde{\mathbb{H}} := \mathbb{F} \otimes \mathbb{G}^N \subseteq \mathbb{H} := \mathbb{F} \otimes \mathbb{G}$.

The financial market, on which the insurance company has full knowledge, consists of a riskless asset with (discounted) price identically equal to 1 and a risky asset whose (discounted) price S is represented by a semimartingale satisfying the so-called *structure condition*, see (2.1). Since the insurance company's decisions are based on the information flow $\widetilde{\mathbb{H}}$, we will look for admissible investment strategies $\psi = (\theta, \eta)$, where the process θ, which describes the amount of wealth invested in the risky asset, is supposed to be $\widetilde{\mathbb{H}}$-predictable, whereas the process η, providing the component invested in the riskless asset, is $\widetilde{\mathbb{H}}$-adapted.

We consider two basic forms of insurance contracts, so-called *pure endowment* and *term insurance*. The policy-holder of a *pure endowment* contract receives the payoff ξ of a contingent claim at a fixed time T, if she/he is still alive at this time, while the *term insurance* contracts state that the sum insured is due immediately upon death before time T. Precisely, payments can occur at any time during $[0, T]$ and are assumed to be time dependent of the form $g(t, S_t)$. In this case the generated obligations are not contingent claims at a fixed time T, however they can be transformed into general *T-claims* by deferring the payments to time T. Under suitable assumptions the payoff of the resulting *insurance claim*, denoted by G_T, in both cases is a square-integrable $\widetilde{\mathcal{H}}_T$-measurable random variable and since the traded asset S turns to be $\widetilde{\mathbb{H}}$-adapted, we can write the Föllmer–Schweizer decomposition of the random variable G_T with respect to S and $\widetilde{\mathbb{H}}$. By applying the results of Schweizer (2001) and Proposition 5.9 we characterize the pseudo-optimal strategy as the integrand in the Föllmer–Schweizer decomposition and the optimal value process as the conditional expected value of the insurance claim G_T with respect to the *minimal martingale measure*, given the information flow $\widetilde{\mathcal{H}}_t$.

In particular, we furnish an explicit formula for the pseudo-optimal strategy in terms of the $\mathbb{G}^N$-projection of the survival process, defined in our framework, as ${}_tp_s := \mathbf{P}_2\big(T_i > s+t \mid \{T_i > s\} \cap \mathcal{G}^X_\infty\big)$. In a Markovian setting, its $\mathbb{G}^N$-optional projection, ${}_t\widehat{p}_s$, can be written by means of the filter π, that provides the conditional law of the stochastic factor X given the observed history $\mathbb{G}^N$. As a consequence, the computation of the optimal strategy and of the optimal value process leads to a filtering problem with point process observations.

The literature concerning filtering problems is quite rich, and in particular we can distinguish three main subjects related to different dynamics of the observation process: continuous, counting and mixed type observations. Counting type observation, which is that considered also in this paper, has been analyzed by Ceci and Gerardi (1997) in the framework of branching processes, and by Ceci and Gerardi (2000, 2001) for pure jump state processes. An explicit representation of the filter is obtained in Ceci and Gerardi (2006) by the Feynman–Kac formula using the linearization method introduced by Kliemann et al. (1990). We use this technique to achieve a similar result in our context. For completeness we indicate some references concerning the continuous case, Kallianpur (1980), Kurtz and Ocone (1988) and Lipster and Shiryaev (2000), and more recently the mixed type observation has been studied in Ceci and Colaneri (2012, 2014), Frey and Schmidt (2012) and Grigelionis and Mikulevicious (2011).

The paper is organized as follows. In Section 2 we describe the financial market model. Section 3 is devoted to the insurance market model. In Section 4 we introduce the combined financial-insurance model. The local risk-minimization is discussed in Section 5. The Föllmer–Schweizer decomposition and explicit formulas for the optimal strategy for both *pure endowment* and *term insurance* contracts are contained in Sections 6 and 7, respectively. Finally, the computation of the survival process and some other technical results are gathered in the Appendix.