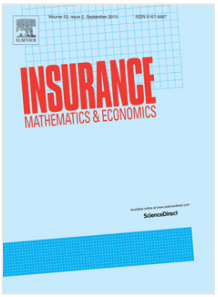

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A Directional Multivariate Value at Risk

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Abstract

In economics, insurance and finance, value at risk (VaR) is a widely used measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, time horizon, and probability α, the $100\alpha\%$ VaR is defined as a threshold loss value, such that the probability that the loss on the portfolio over the given time horizon exceeds this value is α. That is to say, it is a quantile of the distribution of the losses, which has both good analytic properties and easy interpretation as a risk measure. However, its extension to the multivariate framework is not unique because a unique definition of multivariate quantile does not exist. In the current literature, the multivariate quantiles are related to a specific partial order considered in $\mathbb{R}^n$, or to a property of the univariate quantile that is desirable to be extended to $\mathbb{R}^n$. In this work, we introduce a multivariate value at risk as a vector-valued directional risk measure, based on a directional multivariate quantile, which has recently been introduced in the literature. The directional approach allows the manager to consider external information or risk preferences in her/his analysis. We derive some properties of the risk measure and we compare the univariate *VaR* over the marginals with the components of the directional multivariate *VaR*. We also analyze the relationship between some families of copulas, for which it is possible to obtain closed forms of the multivariate *VaR* that we propose. Finally, comparisons with other alternative multivariate *VaR* given in the literature, are provided in terms of robustness.

Keywords: multivariate risks, value at risk, directional approach.
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