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Time-consistent investment strategy under partial information[☆]

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HIGHLIGHTS

- A time-inconsistent investment strategy under partial information is studied.
- Closed-form expressions of equilibrium strategy and value function are provided.
- The results of complete information are also given for the sake of comparison.
- Some numerical examples are given to illustrate our results.

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ABSTRACT

This paper considers a mean–variance portfolio selection problem under partial information, that is, the investor can observe the risky asset price with random drift which is not directly observable in financial markets. Since the dynamic mean–variance portfolio selection problem is time inconsistent, to seek the time-consistent investment strategy, the optimization problem is formulated and tackled in a game theoretic framework. Closed-form expressions of the equilibrium investment strategy and the corresponding equilibrium value function under partial information are derived by solving an extended Hamilton–Jacobi–Bellman system of equations. In addition, the results are also given under complete information, which are need for the partial information case. Furthermore, some numerical examples are presented to illustrate the derived equilibrium investment strategies and numerical sensitivity analysis is provided.

1. Introduction

How to allocate wealth among investment opportunities with different risk characteristics is a central problem in financial economics. It is well known that the mean–variance approach, proposed by Markowitz (1952), is viewed as the foundation of modern finance theory. In the mean–variance portfolio model, the risk is quantified by using the variance, therefore, specifying their acceptable risk level, the investor can seek the highest return. After Markowitz's pioneering work, the mean–variance model is extended by many scholars in subsequent decades. Among others, the representative works include Li and Ng (2000) and Zhou and Li (2000), who extend the originally mean–variance problem to dynamic discrete-time setting and continuous-time setting, respectively. In Li and Ng (2000), the authors introduce an embedding technique to embed the original problem into a tractable auxiliary problem, then the analytical portfolio strategy and mean–variance efficient frontier are derived. Zhou and Li (2000) further consider a continuous-time mean–variance portfolio selection problem. The further study about dynamic extension of the Markowitz's model includes, e.g., Li et al. (2002), Zhu et al. (2004, 2009), Lim (2004), Bielecki et al. (2005) and Gao et al. (2015), etc.

However, it is well known that, due to the failure of the iterated-expectations property for variance in the mean–variance objectives, the optimal dynamic mean–variance portfolio selection

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problem is time inconsistent in the sense that Bellman optimality principle does not hold. As a result, dynamic programming approach cannot be applied. In fact, in all those works mentioned above, the optimal portfolio strategies are called pre-committed ones. The pre-committed means that the decision-makers at the initial time can commit themselves that, if they can choose a strategy that is optimal at the initial time, and then constrain themselves to abide by it in the future, although the strategy is no longer optimal for the future time. In other words, the pre-committed strategy for the mean–variance portfolio selection model is time inconsistent in that the strategy is only optimal at the initial time but no longer optimal in any remaining time interval.

The concept of time inconsistency is first treated formally by Strotz (1956), where a deterministic Ramsay problem is studied and three approaches to deal with the time inconsistent problem are proposed. Further works along this line include, Peleg and Menahem (1973) who discuss the consumer choice with time inconsistent preferences, and the works provided in Pollak (1968), Goldman (1980), Harris and Laibson (2001), Krusell and Smith (2003), etc. In particular, in recent ten years, a few of scholars pay again much attention to the time inconsistent problem. Ekeland and Lazrak (2006), Ekeland and Pirvu (2008) and Ekeland et al. (2012) study the optimal consumption and investment problems with non-exponential discounting from the game theoretic point of view. The dynamic mean–variance asset allocation problem is studied by Basak and Chabakauri (2010), where a time-consistent strategy is determined by a backward recursion starting from the terminal time. Within the game theoretic framework, Björk and Murgoci (2010, 2014), Björk et al. (2014) develop a general theory of time inconsistent stochastic control problems with various forms of time inconsistent objective functions in a Markovian setting, and derive an extended Hamilton–Jacobi–Bellman (HJB) system of equations to characterize the equilibrium value function and the equilibrium strategy. From a perspective of computation, Wang and Forsyth (2011) study the time-consistent strategy and the pre-commitment strategy of a continuous-time mean–variance asset allocation problem, and develop a numerical scheme to determine the strategy where any type of constraint can be applied to the investment behavior. Czichowsky (2013) considers a time-consistent mean–variance portfolio selection problem under a general semimartingale setting. Bensoussan et al. (2014) study the time-consistent strategies in the mean–variance portfolio selection with short-selling prohibition in both discrete and continuous time settings within the framework developed in Björk and Murgoci (2010, 2014).

Besides, the time inconsistent problems in insurance are also studied by some authors. To our knowledge, the first work about time-consistent investment and reinsurance strategies is studied by Zeng and Li (2011). Then Li et al. (2012) consider the time-consistent investment and reinsurance strategies for an insurer under Heston's stochastic volatility model. Zeng et al. (2013) study an investment and reinsurance problem incorporating jumps for mean–variance insurers. Li and Li (2013) further study time-consistent investment and reinsurance problem under the mean–variance criterion with state-dependent risk aversion.

It is worth noting that, using different ideas and techniques, Cui et al. (2012) propose an alternative way to deal with the time inconsistency in the dynamic mean–variance portfolio selection formulation. By introducing a concept of time inconsistency in efficiency and defining an induced trade-off, the authors first demonstrate that, if investor's wealth is above certain threshold during the investment horizon, their behavior will be irrational under the pre-committed optimal mean–variance portfolio strategy. If the self-financing restriction is relaxed and withdrawal of money out of the market is allowed during the investment process, then an investment strategy, which is strictly better than the pre-committed optimal mean–variance strategy, can be devised. That is, for any given pre-committed efficient mean–variance strategy, a revised strategy which can achieve the same mean–variance pair of the terminal wealth enables the investor to obtain some extra (positive) money with a strictly positive probability during the investment process under some certain probability distribution assumptions.

An important feature in all those works is that it is assumed that the driving Brownian motions are completely observable by an investor, which is an ideal model assumption in reality. In practice, an investor can observe only the stock prices including past and present and he will make his investment decision based on this observable information of the stock prices. This leads to the so-called partial information portfolio selection problem. The portfolio selection with partial information has been studied in some literature, for example, Gennotte (1986), Lakner (1998), Xia (2001), Nagai and Peng (2002), Brendle (2006), Björk et al. (2010), Fouque et al. (2015) within the expected utility framework. For the mean–variance model, Pham (2001) considers a mean–variance hedging problem under partial observation, in which the investor observes just the stock prices and wants to price and hedge a contingent claim. Xiong and Zhou (2007) concern with a continuous-time partially observed mean–variance portfolio selection problem. They give a very direct and short proof for a separation principle and employ the particle system representation to develop analytical and numerical approaches in obtaining the filter as well as solving the related BSDE.

As pointed out earlier, it is completely impossible that an investor can observe the stock return rate in real world, while he can only make decision based on the information about stock price. Therefore, it is extremely essential to investigate investment problem under partial information. In the realm of mean variance, time inconsistency will appear, so we will study the equilibrium investment strategy to be serious about the time inconsistency under a game theoretic framework. To the best of our knowledge, there is no literature to consider the time-consistent mean–variance investment strategy under partial information, this study is only an initial attempt to assess the effect of partial information to the time-consistent decision-making.

This paper is a contribution to the mean–variance portfolio selection problem under partial information. Instead of pre-committed strategy, our aim is to seek time-consistent strategy for this partially observed mean–variance portfolio selection problem. Specifically, suppose that the financial market consists of a risk-free asset and a risky asset whose price process follows a geometric Brownian motion, however, the appreciation rate of the risky asset price is stochastic and follows a mean reversion process. An investor observes just the stock prices but not the appreciation rate, that is, the only information available to the investor is the stock prices, based on which the investor will make his investment decision. In order to solve this mean–variance problem under partial information, we firstly reduce the partially observable problem to a completely observable problem by the linear filtering theory, then under the framework developed in Björk and Murgoci (2010), the equilibrium investment strategy and the corresponding equilibrium value function for this problem are obtained by means of solving the extended HJB system of equations.

The rest of this paper is organized as follows. In Section 2, we describe the model and some necessary assumptions and formulate the problem within a game theoretic framework. In Section 3, we study a mean–variance portfolio selection problem under complete information, and explicitly derive the solution to this problem which is needed for partial information case. In Section 4, we solve the mean–variance portfolio selection problem under partial information. In Section 5, we present some numerical results and graphs as illustrations. Finally, Section 6 concludes this paper.