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A generic model for spouse's pensions with a view towards the calculation of liabilities

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ABSTRACT

We introduce a generic model for spouse's pensions. The generic model allows for the modeling of various types of spouse's pensions with payments commencing at the death of the insured. We derive abstract formulas for cashflows and liabilities corresponding to common types of spouse's pensions. In particular, we show that our generic model allows for simple modeling of longevity improvements, enabling the calculation of the Solvency II capital requirements related to longevity risk for spouse's pensions.

1. Introduction

The motivation for this paper is the accurate calculation of the liabilities corresponding to the particular type of life insurance policies known as spouse's pensions. In such a policy, payments are made to the spouse upon the death of the insured, in the case where a spouse is present. Many pension funds offer products such as this and have a considerable interest in efficient, practical estimation of their corresponding liabilities to the policyholders. Our main objective in this paper is to develop a flexible modeling framework for estimation of liabilities for spouse's pensions.

Apart from the calculation of liabilities, the forthcoming Solvency II rules from the European Union have led to increased theoretical and practical interest in the calculation of not only the liability, but also the cashflow of insurance policies, meaning the expected rate of payments on the insurance policy in the future. Here, the directive (European Parliament, 2009) states in Article 77.2 on the calculation of technical provisions: *The best estimate shall correspond to the probability-weighed average of future cash-flows*, see also (CEIOPS, 2009) for details on the implications of this. Consistently with the directive requirements, we aim to obtain a modeling framework which enables the calculation of both liabilities and cashflows. One classical setup for such calculations is to let e.g. the health state of the insured be modeled by a continuous-time Markov chain or semi-Markov chain. For papers related to this, see e.g. Hoem (1969); Christiansen (2012); Buchardt et al. (2014) and the references therein. For spouse's pensions, the presence of a future spouse with a priori unknown age excludes the possibility of a simple Markov chain model, and therefore different methods must be applied to obtain expressions for the cashflow of such policies.

Finally, the Solvency II rules also specify the necessity of modeling the longevity risk inherent in life insurance products, meaning the modeling of longevity improvements in populations over time. Specifically, the directive (European Parliament, 2009) states in Article 105.3 that the Basic Solvency Capital Requirement shall be calculated as a combination of capital requirements for several submodules, including *the risk of loss, or adverse change in the value of insurance liabilities, resulting from changes in the level, trend or volatility of mortality rates, where a decrease in the mortality rate leads to an increase in the value of insurance liabilities*

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(longevity risk), see also (CEIOPS, 2009) for CEIOPS' advice for implementation of the Solvency II requirements related to this. It is therefore of interest to obtain models for the calculation of the liability of spouse's pensions in which longevity improvements are included, such that e.g. the mortality benchmark intensities with longevity improvements reported in The Danish Financial Supervisory Authority (2013) can be used when calculating cashflows and liabilities. In the case of spouse's pensions, where the age of the spouse a priori is unknown, the inclusion of longevity improvement presents challenges not present in the case of simple insurance of a single life.

In this article, we develop a generic model for spouse's pensions, and derive expressions for cashflows and liabilities for a wide family of pension products. We also explicitly consider a marked point process model for modeling the health of the insured and the spouse, allowing for longevity improvements for the spouse. In developing this model, we are indebted to the formulas of the Danish G82 concession, where several types of spouse's pensions are described and formulas for the estimation of the corresponding liabilities are presented, see e.g. (Cederbye et al., 1997) for more on this.

The remainder of the article is structured as follows. In Section 2, we review the notions of payment processes, expected cumulative payments, cashflows and liabilities in the context of a simple Markov chain model. In Section 3, we introduce our generic model for spouse's pensions and derive expressions for cashflows and liabilities. In Section 4, we introduce a marked point process model for the insured and the spouse, including longevity improvements. Section 5 contains remarks on numerical methods and some numerical examples. Finally, in Section 6, we discuss our results. Section 7 contains proofs.

2. Review of the continuous-time Markov chain framework

In order to motivate our model, we first recall the modeling framework based on continuous-time Markov chains as discussed in e.g. Buchardt et al. (2014). Consider a simple life insurance product paying one amount of monies per time from a given timepoint c and onwards, for as long as the insured is alive. Let Z be the health state of the insured, taking the values a (alive) and d (dead). In order to model this insurance product, we may consider the process

$$B_t = \int_0^t 1_{(Z_t=a)} 1_{(t\geq c)} \, ds. \tag{1}$$

For any $t \geq 0$, B_t describes the cumulative payments paid out to the insured. We refer to B as the cumulative payments process, or simply as the payment process. We may then consider

$$A_t = EB_t = \int_0^t 1_{(t\geq c)} P(Z_t = a) \, ds, \tag{2}$$

the expected cumulative payments. Since A is continuous and differentiable almost everywhere, we may let a denote the Radon–Nikodym derivative with respect to the Lebesgue measure, yielding

$$a_t = 1_{(t\geq c)} P(Z_t = a). \tag{3}$$

We refer to a as the cashflow corresponding to the insurance policy. Finally, introducing an interest rate model based on a deterministic short rate r, we may define

$$L = E \int_0^\infty e^{-rt} \, dB_t, \tag{4}$$

the liability corresponding to the insurance policy. These concepts of cumulative payment processes, expected cumulative payments, cashflows and liabilities, are well known in various guises from the literature, see e.g. Hoem (1969, 1972); Norberg (1991); Buchardt et al. (2014). In the next section, we use the same framework in the context of a generic model for spouse's pensions.

3. A generic model for spouse's pensions

In this section, we introduce our generic model for spouse's pensions. We are interested in modeling spouse's pensions of the type where the spouse is entitled to certain payments contingent upon the death of the insured as well contingent upon a generic 'policy state'. Usually, this latter 'policy state' will be the health state of the spouse, such that e.g. payments only are made for as long as the spouse is alive, but for the sake of flexibility, we do not limit ourselves as regards the nature of this policy state space. After the introduction of the modeling framework, we derive expressions for cashflows and liabilities in the generic model. Also, we illustrate the usefulness of our model by deriving expressions for cashflows and liabilities for several types of spouse's pensions.

Consider a probability space $(\Omega, \mathcal{F}, P)$. Let T be a random variable taking its values in $\mathbb{R}_+$, describing the time of death for the insured. Let X be a random variable taking the two values x_s and x_m, corresponding to 'single' and 'married', respectively, describing the marital state of the insured at the time of death T. Let Y be a random variable denoting the age of the spouse at the time of death T. Here, we let $\partial \notin \mathbb{R}_+$ be a 'coffin state' held by Y if the insured was unmarried at the time of death, meaning that we assume $Y = \partial$ whenever $X = x_s$, and otherwise Y takes its values in $\mathbb{R}_+$.

Finally, for each $u, y \geq 0$, let $(Z_v^{u,y})_{v\geq u-y}$ denote a stochastic process on $[u-y, \infty)$ with some common finite state space E. We think of $Z^{u,y}$ as a stochastic generic 'policy state' for the case where a spouse exists at the time of death of the insured, and that spouse has age y at time u. Consistently with this, we let $Z^{u,y}$ be defined on $[u-y, \infty)$, where $u-y$ is the (possibly negative) timepoint when the spouse then had age zero. In the most common case, $Z^{u,y}$ will describe the health of the spouse. Furthermore, let $D_v(E)$ denote the space of cadlag functions from $[v, \infty)$ to E, and assume that $Z^{u,y}$ takes its values in $D_{u-y}(E)$. Let **FV** denote the space of mappings of finite variation from $\mathbb{R}_+$ to $\mathbb{R}$. For $u, y \in \mathbb{R}_+$, let the measurable mappings $\Pi_{u,y} : D_{u-y}(E) \to$ **FV** be given, where both spaces are endowed with the σ-algebras induced by the coordinate projections. We refer to the mappings $(\Pi_{u,y})$ as the payment functions, and define a process C by

$$C_t = \Pi_{T,Y,t}(Z^{T,Y}), \tag{5}$$

where $\Pi_{u,y,t}(z) = \Pi_{u,y}(z)_t$. The interpretation of this is as follows. The expression $\Pi_{u,y,t}(z)$ represents the cumulative payments made to the spouse at time t, given that the death of the insured occurred at time u, that the insured was married at that time, and that the age of the spouse at that time was y, and given the policy state history z since the birth of the spouse. As a consequence, C represents the unconditional cumulative payments for the insurance policy with payment functions $(\Pi_{u,y})$, excepting that C does not prescribe payments to begin conditionally upon the death of the insured while married.

It remains to define the actual cumulative payment process corresponding to the components of the spouse's pension described above, similarly to how we defined cumulative payment processes for a simple policy in Section 2. To this end, we define a process B by

$$B_t = \int_0^t 1_{(s\geq T)} 1_{(X=x_m)} \, dC_s. \tag{6}$$

The process B then has paths of finite variation, and corresponds to the cumulative payment process for the spouse's pension with payment functions $(\Pi_{u,y})$. Given the joint distribution of