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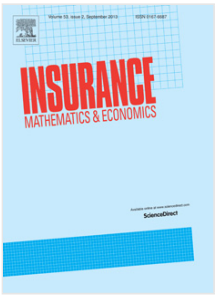

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On the time value of Parisian ruin in (dual) renewal risk processes with exponential jumps

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Abstract

This paper studies the Parisian ruin problem first proposed by Dassios and Wu (2008a,b), where the Parisian ruin time is defined to be the first time when the surplus process has stayed below zero continuously for a pre-specified time length d. Both the insurance risk process and the dual model will be considered under exponential distributional assumption on the jump sizes while keeping the inter-arrival times arbitrary. In these two models, the Laplace transform of the Parisian ruin time is derived by extending the excursion techniques in Dassios and Wu (2008a) and taking advantage of the memoryless property exponential distributions. Our results are represented in integral forms, which are expressed in terms of the (joint) densities of various ruin-related quantities that are available in the literature or obtainable using the Lagrange's expansion theorem. As a by-product, we also provide the joint distribution of the numbers of periods of negative surplus that are of duration more than d and less than d, which can be obtained using some of our intermediate results. The case where the Parisian delay period d is replaced by a random time is also discussed, and it is applied to find the Laplace transform of the occupation time when the surplus is negative. Numerical illustrations concerning an Erlang(2) insurance risk model are given at the end.

Keywords: Parisian ruin time; Sparre Andersen model; Dual risk model; Lagrange's expansion theorem; Excursion; Occupation time in red.

1 Introduction

In this paper, the surplus process $\{U_\cdot(t)\}_{t\geq 0}$ of a business enterprise is generally modelled by

$$U_\cdot(t) = u \pm X(t), \qquad t \geq 0, \tag{1.1}$$

where $U_\cdot(0) = u \geq 0$ is the initial surplus and $X(t) = ct - \sum_{i=1}^{N(t)} Y_i$ with $c > 0$. It is assumed that $\{Y_i\}_{i=1}^{\infty}$ is a sequence of independent and identically distributed (i.i.d.) positive jumps, and $\{N(t)\}_{t\geq 0}$ is a renewal process that is independent of $\{Y_i\}_{i=1}^{\infty}$ and characterized by the sequence of i.i.d. inter-arrival times $\{V_i\}_{i=1}^{\infty}$. In the rest of the paper, it is assumed that each Y_i is exponentially distributed with

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