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Pricing annuity guarantees under a double regime-switching model

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ABSTRACT

This paper is concerned with the valuation of equity-linked annuities with mortality risk under a double regime-switching model, which provides a way to endogenously determine the regime-switching risk. The model parameters and the reference investment fund price level are modulated by a continuous-time, finite-time, observable Markov chain. In particular, the risk-free interest rate, the appreciation rate, the volatility and the martingale describing the jump component of the reference investment fund are related to the modulating Markov chain. Two approaches, namely, the regime-switching Esscher transform and the minimal martingale measure, are used to select pricing kernels for the fair valuation. Analytical pricing formulas for the embedded options underlying these products are derived using the inverse Fourier transform. The fast Fourier transform approach is then used to numerically evaluate the embedded options. Numerical examples are provided to illustrate our approach.

1. Introduction

Equity-linked annuities (ELAs) are one of the major innovations in the insurance industry. They provide policyholders with insurance protection as well as investment returns from equity markets. These contracts allow the flexibility to provide both life insurance benefits and guaranteed minimum accumulation benefits. Typically, in an ELA contract, an insurer will make periodic payments to the beneficiary, while the policyholder pays a lump-sum premium at the initiation of the contract. From a policyholder's perspective, ELAs provide minimum guarantees on downside risk and upside potential profits. The policyholder is also provided with the flexibility to select the composition of an investment portfolio. Furthermore, the tax-deferred feature is another advantage of these products. From the perspective of an insurer, higher insurance fees is a main advantage. In practice, the operational procedure to sell ELAs is comparatively easier. These may explain why both policyholders and insurers prefer these products to other long-term investments with lower yields, including bank accounts, bonds, and so on. Two popular types of ELAs are equity-indexed annuities (EIAs) and variable annuities (VAs) with various embedded guarantees.

The valuation of ELAs, including EIAs and VAs has attracted a considerable interest from both academic researchers and market practitioners. The literature mostly investigate the valuation of ELAs based on the interplay between an option and an ELA (see Boyle and Schwartz, 1977 and Brennan and Schwartz, 1976, 1979). The guaranteed minimum benefit can be viewed as a kind of embedded options. Much attention has been given to the EIAs valuation under the Black–Scholes framework, including Tiong (2000), Lee (2003), etc. Lin and Tan (2003) and Kijima and Wong (2007) investigated the valuation of EIAs with stochastic interest rates and mortality risk, while Qian et al. (2010) considered the EIAs valuation with stochastic mortality rate. Milevsky and Posner (2001) investigated the valuation of guaranteed minimum death benefit (GMDB) in VAs by the risk-neutral pricing theory. Examples of considering the valuation of guaranteed minimum withdrawal benefit (GMWB) in VAs include Milevsky and Salisbury (2006) and Dai et al. (2008). Hardy (2003) presented an overview of various investment guarantees. Bauer et al. (2008) considered a general pricing framework for all types of guarantees in VAs. Siu et al. (2007) and Ng et al. (2011) discussed the valuation of investment guarantees under GARCH-type models.

Regime-switching models are popular and practically useful models in econometrics and finance. This class of models was

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popularized by Hamilton (1989) to economics. One of the main advantages of regime-switching models is that they allow the flexibility to describe the impacts of the structural changes in economic conditions. Typically, the model parameters are assumed to change over time with the states of a modulating Markov chain, representing the states of an underlying economy. Recently, regime-switching models have been applied to various practical problems in finance and insurance. A major area of these applications is option valuation and its counterpart in insurance, namely the fair valuation of insurance products. Due to the very nature of the ELAs such as the long-term maturity, the use of regime-switching models to evaluate fair values of liabilities underlying ELAs is deemed to be practically relevant. Hardy (2001) discussed the valuation and hedging of long-term investment guarantees under regime-switching models. Some examples of pricing participating life insurance products under regime-switching models include Siu (2005) and Siu et al. (2008). Lin et al. (2009) discussed the valuation of EIAs and VAs under a regime-switching model under the assumption that the model dynamics of the reference investment fund value is a geometric Brownian motion modulated by a continuous-time, finite-state Markov chain. Yuen and Yang (2010) applied the trinomial tree method to value EIAs with regime-switching. Ng and Li (2011) first studied the valuation of variable annuity guarantees under a multivariate regime-switching model. Qian et al. (2012) considered the valuation of equity-indexed annuities with regime-switching jump-diffusion model and stochastic mortality, where the jump-component is described by a compound Poisson process and is independent of the modulating Markov chain. So jumps in the share price may not be triggered by state transitions. Fan (2013) considered the valuation of variable annuities with GMDB when the investment fund follows a Markov-modulated geometric Brownian motion.

However, most of the existing literature on financial products valuation under regime-switching models suppose that only the model parameters change according to the modulating Markov chain. Comparatively little attention has been paid to regime-switching models with the assumption that both model parameters and the price level of the reference investment fund may change when the modulating Markov chain switches from one state to another. Using the terminology in Shen et al. (2014), when a regime switch occurs, the model with only model parameters changing is called the single regime-switching (SRS) model, while the other kind of regime-switching model is denoted as the double regime-switching (DRS) model. The DRS models are more flexible than their SRS counterparts to describe the stochastic movements of the reference investment fund due to the fact that a jump in the investment fund price level occurs in the former, rather than the latter, when there is a regime switch (see Naik, 1993, Yuen and Yang, 2009, Elliott and Siu, 2011, Elliott et al., 2011 and Shen et al., 2014).

In this paper, we adopt the DRS model proposed in Shen et al. (2014) to investigate the valuation of the equity-linked products with mortality risk. Various designs including the point-to-point EIAs, the annual ratchet EIAs and the GMDB in variable annuities are considered. The main feature of the DRS model is that it provides a way to endogenously determine the regime-switching risk, while the regime-switching risk is either ignored or taken exogenously in earlier works. More specifically, the model parameters, including the risk-free interest rate, the appreciation rate and the volatility, are modulated by a continuous-time, finite-state, observable Markov chain. Meanwhile, the price level of the reference investment fund may experience a jump when a regime switch occurs. The martingale, associated with the modulating Markov chain, is used to describe the jump component of the reference investment fund value. This describes quantitatively jumps in the price level when regime switches occur. Here, we present two approaches to selecting pricing kernels. Firstly, we use the generalized version of the regime-switching Esscher transform introduced in Shen et al. (2014) to select an equivalent martingale measure (i.e. risk-neutral probability measure). Then we discuss the selection of an equivalent martingale measure using the minimal martingale measure method. Both approaches allow us to determine a unique equivalent martingale measure and incorporate not only the diffusion risk described by the Brownian motion but the regime-switching risk (or the jump risk) modeled by the Markov chain in the valuation. Under the selected risk-neutral probability measure, we use the inverse Fourier transform to derive integral pricing formulas for the embedded options. The fast Fourier transform (FFT) method is adopted to discretize the integral pricing formulas. Since the double regime-switching model is an extension of the single one, the valuation problem under the single model considered in Fan (2013) may be considered as a particular case of the valuation problem in our current paper. Using the FFT method, we provide the numerical examples to illustrate the valuation of the point-to-point EIAs and GMDB in VAs under both the double regime-switching model and the single regime-switching model as well as document the pricing implications of these two models.

Lin et al. (2009), one of our main references, considered an interesting problem to price annuity guarantees under a regime-switching model. Our paper extends the results of Lin et al. (2009) in the following aspects. Firstly, we consider the valuation of EIAs and VAs under a double regime-switching model in Shen et al. (2014). In addition to the assumption that model parameters are governed by the modulating Markov chain adopted in Lin et al. (2009), we also assume that a jump in the price level of the reference investment fund may occur when the modulating Markov chain switches from one state to another. In other words, the impacts of the regime-switching risk were not considered in Lin et al. (2009). However, the regime-switching risk brought by the state transitions of the underlying economy is difficult, if not impossible, to be diversified. This may suggest that the regime-switching risk may not be ignored. Secondly, we provide two different ways to endogenously determine the regime-switching risk using the generalized regime-switching Esscher transform and the minimal martingale measure approach. In the discussion part of Lin et al. (2009), an exogenous way to quantify the regime-switching risk was provided. However, there may exist more than one solutions for the regime-switching Esscher transform parameters from the given density process described in the discussion of Lin et al. (2009). Other techniques are needed to choose an equivalent martingale measure. In the model we considered here, without imposing other criteria or constraints, a unique pricing kernel can be selected using either the generalized version of regime-switching Esscher transformation or the minimal martingale measure approach. Furthermore, this pricing kernel also provides a quantification for the regime-switching risk. Thirdly, our results may be easier to be extended to a multi-regime case. The analytical pricing formulas, obtained via the FFT approach, look quite neat and the convergence rate of the FFT approach is reasonably fast.

The rest of the paper is organized as follows. The next section presents the model dynamics. In Section 3, we select equivalent martingale measures using the generalized version of the regime-switching Esscher transformation and the minimal martingale measure approach. Section 4 presents the valuation of the point-to-point EIAs and the annual ratchet EIAs. The FFT approach and the Monte Carlo method are applied to calculate the prices of the point-to-point EIAs and the annual ratchet EIAs, respectively. The valuation of the variable annuities with GMDB is considered in Section 5. In Section 6, we give numerical examples to illustrate the valuation of the point-to-point EIAs, the annual ratchet EIAs and VAs with GMDB. Section 7 concludes the paper.