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On rational pricing for a profit-seeking insurer in the year of hard market

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ABSTRACT

The aim of this paper is to examine rational pricing of a profit-seeking insurer carrying on its business when underwriting cycle is in its upper phase. We focus on migration of insureds wishing to get the same services at a lower price. We investigate pricing which maximizes the insurer's intrinsic value linked to its attractiveness for investors, provided that its solvency position is fixed. The main tool in this paper is explicit bounds on ruin capital in Lundberg risk model with migration. Written in terms of elementary functions, they make the solution straightforward.

1. Introduction

This paper contributes to endogenous explication of the underwriting cycles. It is a step forth in building an integral mathematical model of the cycles from the premises of the theory of reflexivity.[1] It develops Expansion Revenue Solvency (ERS) analysis put forth in Malinovskii (2013a), as a quantitative tool of the modeling. In a nutshell, it emphasizes the need of regular inspecting of the company's size, profit and solvency positions all together rather than of the first two of them. Being a many-factors approach, ERS analysis complies with the views of many experts (see, e.g., Lai et al., 2000).

Though embedded in the problem to explore the nature of the cycles, our view is limited to one year in the cycle's upper phase.[2] We assume that the market is free from fierce price competition.[3] We consider a profit-seeking and incumbent insurer which plays against non-antagonistic contenders. It complements the research outlined in Malinovskii (submitted manuscript), where we examined the difference in pricing among small and large firms, in Malinovskii (2014b), where we modeled how an aggressively expanding insurance company becomes insolvent, and in Malinovskii (in press), where we dealt with a profit-seeking insurer operating under deficiency of information. This deficiency is unavoidable in the stage of advanced competition within the cycle's upper phase.

From the standpoint of the theory of reflexivity applied in Malinovskii (2013a) to model the insurance cycles, the case of a profit-seeking company on the profitable market free from fierce price competition differs from all these three cases. The essence of the difference is that reflexivity hallmarked by a double-feedback mechanism is so feeble that it can be safely ignored.[4] From the

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[1] By reflexivity we mean a discrepancy between the participants' cognition and the situation in which they participate. It is a double-feedback mechanism which is always at work in the competitive business, but which may manifest itself with different force. Introduced in Soros (1994), it was discussed by many authors. Beinhocker (2013) provides updated references, gives more precise definition of fundamental concepts related to reflexivity, and claims that "economics needs to recognize that it made an ontological error when in the nineteenth century it categorized economies as equilibrium systems" and that "the field needs to embrace the economy in its full messy, uncertain, disequilibrium, complex reflexive reality".

[2] Such year, when the market price exceeds the marginal cost of insurance, is called the year of hard, or profitable market.

[3] This situation may change abruptly, as an aggressive firm enters the market.

[4] Quoting from Preface of Soros (1994), "In *The Alchemy of Finance*, I put forward the theory of reflexivity as if it were relevant at all times. That is true in the sense that the two-way feedback mechanism that is the hallmark of reflexivity can come into play at any time, but it is not true in the sense that it is at play at all times. In fact, in most situations it is so feeble that it can be safely ignored. We may distinguish between near-equilibrium conditions where certain corrective mechanisms prevent perceptions and reality from drifting too far apart, and far-from-equilibrium conditions where a reflexive double-feedback mechanism is at work and there is no tendency for perceptions and reality to come close together without a significant change in the prevailing conditions, a change of regime.

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standpoint of modeling, it would mean that we act in a situation where the market price is well predictable.

Though we restrict ourselves to a year of profit and peace, it is competition and striving for individual success which always remain the main business engine. It is commonly recognized that an adequate pricing keeps the customers loyal, while rate cuts below the market price attract new customers, and pricing above the market price repulses clients. Alike, stable and high profit attracts new shareholders and retains those who have already bought shares.[5] Since the policyholders are the business customers and shareholders are its owners, the insurance manager ought to pay attention to these relationships.

Developing rational pricing in a year of profit and peace, one has to reckon with many objectives rather than to be aimed only at the company's revenue. One must comply with the solvency constrains and maintain insureds' migration in a balance. Important is to measure these business aspects correctly. Measuring solvency in a tangible way,[6] we focus on the ruin capital set to keep the annual probability of ruin at a preselected level. Dealing with the company's attractiveness for investors, we focus on its intrinsic value defined in Malinovskii (2014a) as the ratio of its mean annual revenue to the ruin capital. The fluctuations in the portfolio volume due to migration of insureds seeking for better prices are modeled by means of rationally chosen migration rate functions.

Addressing the problem of finding prices that maximize the company's intrinsic value in the years of hard market, we apply the upper bounds on ruin capital and lower bounds on the company's intrinsic value, both regarded as functions of individual price. These bounds were obtained in an explicit form in Malinovskii (2014a). Based on the structure of these bounds, we divide the problem in two stages. The first stage is to split the price interval[7] $[\mathsf{E}Y, \infty)$ into a series of successive sub-intervals. It is done by solving an explicitly written equation. The second stage consists in maximization of the intrinsic value within these sub-intervals. If migration rate functions are given, both these stages turn into simple exercises. Both are easily done by means of elementary mathematics. We describe the algorithm of their solution and illustrate it by numerical calculations.

In accordance with the basic requirement of ERS analysis, we further examine whether that price which maximizes the company's intrinsic value may also be called a rational choice from the standpoint of business expansion and solvency. We provide illustrative examples which show that it is so on a high-profitable market, while it is not so, or is not so apparent, on a low-profitable market.

The rest of the paper is arranged as follows. In Section 2, we give basic definitions and present the bounds on annual ruin capital and annual company's intrinsic value obtained in Malinovskii (2014a).

In Section 3, we address selection of prices that maximize the company's intrinsic value. To illustrate the main analytical formulas, we draw Figs. 1–18. They demonstrate that a rationally chosen premium price for a profit-seeking insurer in a year of hard market depends heavily on

- how much the market price is higher than the marginal cost of insurance, i.e., how profitable is this market,
- how sensitive are the policyholders to the company's prices.[8]

In Section 4, we summarize our conclusions. We examine how the market's profitability and the shape of migration rate functions affect the conclusion that the price maximizing the company's intrinsic value is rational. The analysis of Figs. 1–18 shows that when the value of ruin capital which corresponds to this price is large compared to other values of ruin capital, as price ranges within the interval of policyholders' sensitivity to price variation, it may be nonrational. Being acceptable from the standpoint of business attractiveness and expansion, it may fail from the standpoint of solvency.

In Section 5, we report several auxiliary results.

2. Basic definitions and fundamental inequalities

The definitions in this section largely follow the lines of Malinovskii (2013b, 2014a,b, submitted manuscript, in press). We present them here for completeness.

2.1. Migration of insureds and portfolio volume

The first step in modeling migration of insureds who seek for better prices, is to set migration rate functions. They measure changes in time of a unit of the portfolio volume, as a number of factors affect these changes. The following general definition was introduced in Malinovskii (2013b).

Definition 1 (*Migration Rate Functions*)**.** Assuming that $\varrho > 0$ is a fixed market price, we call migration rate functions the family of functions $r_s(p \mid \varrho, l)$, where $p > 0$ is the company's price, $s \geqslant 0$ is time, and $l \geqslant 0$ is migration intensity factor, as they satisfy the following conditions. They are identically 1, as $s = 0$, or[9] $p/\varrho = 1$, or $l = 0$. For $s > 0$ and p fixed, the function $r_s(p \mid \varrho, l)$ of the variable $l > 0$

- is 1, as $l = 0$, and is monotone increasing, as l increases, provided that $0 < p/\varrho < 1$; while increasing, it remains bounded from above by c_U,
- is 1, as $l = 0$, and is monotone decreasing, as l increases, provided that $p/\varrho > 1$; while decreasing, it remains bounded from below by c_L.

For $s > 0$ and $l > 0$ fixed, the function $r_s(p \mid \varrho, l)$ of the ratio p/ϱ (or of price p)

- is monotone decreasing, as p/ϱ decreases, being bounded from above by c_U and from below by c_L; while decreasing, it passes through 1, as $p/\varrho = 1$.

For $l > 0$ and $p/\varrho > 0$ fixed, the function $r_s(p \mid \varrho, l)$ of time s

In the first case, classical economic theory applies and the divergence between perceptions and reality can be ignored as mere noise. In the second case, the theory of equilibrium becomes irrelevant and we are confronted with a one-directional historical process where changes in both perceptions and reality are irreversible. It is important to distinguish between these two different states of affairs because what is normal in one is abnormal in the other". Quoting further on, "I now believe this point", i.e. biased views of individual participants and their dismiss as temporary aberrations, "can be more effectively made by drawing a distinction between near-equilibrium and far-from-equilibrium conditions than by proposing a general theory of history based on the constant cross-crossing between perceptions and reality as I have done in *The Alchemy of Finance*. That does not mean that there is anything wrong with the general theory; it means only that the concept of reflexivity becomes more significant if it is reserved for those cases where the double feedback mechanism is actually at work".

[5] In the long run, the shareholders may be pleased with good prospects of their equity insurance business, i.e. its solvency and growth. But in the short run, they always wish profit.

[6] The probability of ruin is a dimensionless factor. It may embarrass some managers accustomed to the factors measured in money units, such as reserves and prices. Though the definition of the ruin capital is more complex than of the probability of ruin, its use is straightforward.

[7] By $\mathsf{E}Y > 0$ we denote the marginal cost of insurance. See Section 2.2 for accurate definitions.

[8] Importance of this factor is well known to practitioners. Quoting Daykin et al. (1996, p. 343), "one of the relevant factors is the price sensitivity of policyholders. This obviously depends on the extent to which brokers are used and can be very different for commercial policies and personal lines policies".

[9] Note that, as ϱ is fixed, it is convenient to move from p to the ratio $d = p/\varrho > 0$.