Insurance: Mathematics and Economics 62 (2015) 118–137

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Optimal investment–reinsurance strategy for mean–variance insurers with square-root factor process[☆]

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HIGHLIGHTS

- A mean–variance investment–reinsurance problem is considered.
- Both Markovian and non-Markovian structures are included in our modeling framework.
- The solvability of two BSDEs with unbounded parameters is proved.
- Closed-form solutions for efficient strategy and efficient frontier are derived.
- Two special cases are discussed and further demonstrated by numerical examples.

ARTICLE INFO

Article history:
Received November 2014
Received in revised form January 2015
Accepted 10 March 2015
Available online 18 March 2015

MSC:
IM52
IE13
IB91

Keywords:
Investment–reinsurance
Mean–variance criterion
Backward stochastic differential equation
Efficient strategy
Efficient frontier

ABSTRACT

This paper studies an optimal investment–reinsurance problem for an insurer with a surplus process represented by the Cramér–Lundberg model. The insurer is assumed to be a mean–variance optimizer. The financial market consists of one risk-free asset and one risky asset. The market price of risk depends on a Markovian, affine-form, square-root stochastic factor process, while the volatility and appreciation rate of the risky asset are given by non-Markovian, unbounded processes. The insurer faces the decision-making problem of choosing to purchase reinsurance, acquire new business and invest its surplus in the financial market such that the mean and variance of its terminal wealth is maximized and minimized simultaneously. We adopt a backward stochastic differential equation approach to solve the problem. Closed-form expressions for the efficient frontier and efficient strategy of the mean–variance problem are derived. Numerical examples are presented to illustrate our results in two special cases, the constant elasticity of variance model and Heston's model.

1. Introduction

Traditionally, reinsurance is an effective tool for insurers to protect them against large losses. On the other hand, insurers are now playing an active role in the financial market. Indeed, investment has become an indispensable part of the insurance business. These two aspects have facilitated tremendous research interests on optimal investment–reinsurance problems in recent years. For example, Browne (1995), Irgens and Paulsen (2004), Yang and Zhang (2005), Zhang and Siu (2012) and Liang and Bayraktar (2014) study the optimal investment and/or reinsurance strategies in the sense of maximizing the expected utility from the terminal wealth of insurers with different market assumptions. Hipp and Plum (2000), Schmidli (2002), Promislow and Young (2005), Chen et al. (2010), Luo and Taksar (2011) and Azcue and Muler (2013) focus on seeking the optimal investment and/or reinsurance strategies to minimize the ruin probability of insurers with different constraints and market assumptions. Bäuerle (2005), Delong and Gerrard (2007), Bai and Zhang (2008), Zeng et al. (2010), Zeng and Li (2011), Chiu and Wong (2012), Chen and Yam (2013), Zeng et al. (2013), Bi et al.

☆ We are very grateful to the editor and an anonymous referee for their valuable comments. This research is partially supported by grants of the National Natural Science Foundation of China (Nos. 71231008, 71201173, and 71301031), Natural Science Foundation of Guangdong Province of China (No. S2013010011959) and Research Funds for the Central Universities (No. 13wkpy28).

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http://dx.doi.org/10.1016/j.insmatheco.2015.03.009
0167-6687/

(2014) and Shen and Zeng (2014) investigate the optimal investment and/or reinsurance strategies in different situations under the mean–variance criterion.

The risky asset prices in most of the aforementioned works are assumed to have constant or deterministic volatilities. However, many empirical studies show stochastic volatility (SV) does exist in the financial market. See French et al. (1987), Pagan and Schwert (1990), amongst others. Nowadays, SV has been widely accepted as an important feature in asset price models, and can explain many well-known empirical facts, such as heavy-tailed nature of return distributions, volatility clustering, volatility smile, etc. In addition, many scholars have considered optimal investment and/or consumption problems with the risky asset prices following SV models. See Zariphopoulou (1999), Chacko and Viceira (2005), Liu (2007), Noh and Kim (2011), Zeng and Taksar (2013), and references therein.

Recently, optimal investment–reinsurance problems for insurers in the presence of stochastic volatility has attracted some attention. Gu et al. (2010) consider the optimal investment-proportional reinsurance strategy for an insurer to maximize the expected exponential utility of the terminal wealth, where the surplus process is approximated by a diffusion model and the risky asset price is described by a constant elasticity of variance (CEV) model. Lin and Li (2011) extend Gu et al. (2010) to the case of a jump-diffusion surplus model. Gu et al. (2012) extend Gu et al. (2010), in another direction, to the case that the insurer is allowed to purchase excess-of-loss reinsurance. Li et al. (2012) investigate the optimal time-consistent investment-proportional reinsurance strategy for an insurer under the mean–variance criterion, where the risky asset price satisfies Heston's model. Zhao et al. (2013) study the optimal excess-of-loss reinsurance–investment strategy for an insurer to maximize the expected utility of the terminal wealth, where the insurer's surplus is described by a jump-diffusion model and the risky asset price follows Heston's model. Yi et al. (2013) consider a robust optimal investment-proportional reinsurance problem under the utility maximization criterion and Heston's model. Among these works, most adopt the utility maximization criterion and only Li et al. (2012) use the mean–variance criterion. However, Li et al. (2012) only derive the optimal time-consistent strategy and it is a pity that the optimal strategy is deterministic and independent of the current wealth. To our knowledge, there is no literature on the precommitment (globally optimal) strategy of the optimal investment–reinsurance problem with SV model under the mean–variance criterion.

In this paper, we pioneer the study on the precommitment (globally optimal) strategy for a mean–variance insurer's optimal investment–reinsurance problem in a more general model. Specifically, the surplus process of the insurer is described by the classical Cramér–Lundberg model. The insurer can purchase proportional reinsurance/acquire new business and invest its surplus in a financial market consisting of one risk-free asset and one risky asset. The market price of risk is assumed to depend on a stochastic factor, which satisfies an affine-form, square-root, Markovian model. In the previous literature on the optimal investment–reinsurance problems for insurers, the price processes of the risky assets are usually assumed to satisfy some specific Markovian models. On the contrary, it is not a prerequisite to specify the structures of the appreciation rate and volatility processes in our paper and indeed, they are general non-Markovian, unbounded stochastic processes. This reflects the generality of our modeling framework. The general framework makes the geometric Brownian motion, CEV model and Heston's model as special cases. Moreover, some asset price models with non-Markovian appreciate rate and volatility are also included in our framework. The insurer's objective is to seek an optimal investment–reinsurance strategy to maximize the expected terminal wealth and minimize the variance of the terminal wealth simultaneously. Indeed, the insurer's mean–variance problem is a bi-objective stochastic optimization problem. We first transform the bi-objective problem to a constrained variance-minimization problem. Then by the well-known Lagrangian duality method, we relate the constrained variance-minimization problem to an equivalent min–max problem with a quadratic cost functional. To use backward stochastic differential equations (BSDEs) to solve our problem, we discuss the solvability of a backward stochastic Riccati equation (BSRE) and a linear BSDE. Since the market price of risk process depends on an unbounded square-root factor, the existing results for BSDEs are not applicable in our paper directly. Under some exponential integrability assumptions (Novikov's condition and Kazamaki's condition) on the market price of risk, we use some measure change techniques to prove that the corresponding BSRE and linear BSDE admit unique solutions and obtain their closed-from solutions. Based on the unique solutions of BSDEs, we derive closed-form expressions for the efficient frontier and efficient strategy of our mean–variance problem. Moreover, we consider two special cases of our model, namely, the CEV model and Heston's model. Closed-form expressions for efficient frontiers and efficient strategies in the two cases are also provided. Finally, we give some numerical examples to illustrate our results, and show the effect of model parameters on the efficient frontiers in the CEV model and Heston's model.

In one recently published paper, Shen et al. (2014), a mean–variance portfolio selection problem under the CEV model is considered. The current paper differentiates from Shen et al. (2014) at least in four aspects. First of all, the assumptions in the current paper are much weaker than those in Shen et al. (2014). Indeed, we require that the market price of risk process satisfies Novikov's condition and Kazamaki's condition of orders $\frac{1}{2}$ and 2, respectively, while Shen et al. (2014) require the market price of risk process is exponentially integrable up to order $5(3+\sqrt{21})$ (see Remark 3.1 in Shen et al. (2014)). Secondly, the underlying models are different. The financial market in Shen et al. (2014) is complete, while that in the current paper may be incomplete. This makes the mean–variance problem in the current paper more challenging. Thirdly, the induced BSDEs are different due to different market structures. Although a BSRE is also considered in Shen et al. (2014), it can be transformed to a linear BSDE by using Itô's formula directly (see also Shen (2015)). The BSRE associated with the incomplete market in the current paper can be only transformed to a quadratic BSDE, which is much more complicated than the linear BSDE, particularly when model parameters are unbounded. Fourthly, the optimal strategies are obtained in different spaces. The optimal strategy in the current paper is established in a locally square-integrable space. However, the optimal strategy in Shen et al. (2014) indeed lives in a smaller space since the BSDE therein is proved to admit a unique solution in a space accommodating stochastic Lipschitz coefficients, which is smaller than the usually used square-integrable solution space for BSDEs. Therefore, the admissible strategy in Shen et al. (2014) needs to be defined with much stronger integrability than that in the current paper.

Our paper is also different from one recent paper, Chiu and Wong (2014), where also a precommitment mean–variance problem for an insurer is considered taking into account the effect of SV. Besides the different underlying models for SVs (high-dimensional, Markovian model vs. one-dimensional, non-Markovian one), the research topics, BSDE techniques, and most importantly admissible strategies in Chiu and Wong (2014) and the current paper are different. Firstly, Chiu and Wong (2014) consider an asset–liability management problem, while we investigate an optimal investment–reinsurance problem. The essential difference is that the insurance business is controllable by adjusting the proportional reinsurance cover in our paper, while that is uncontrollable in Chiu and Wong (2014). Secondly, the existence and uniqueness of relevant BSREs are proved in Chiu and Wong (2014) via using some