Accepted Manuscript

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Please cite this article as: Yao, K., Qin, Z., A modified insurance risk process with uncertainty. *Insurance: Mathematics and Economics* (2015),
http://dx.doi.org/10.1016/j.insmatheco.2015.03.029

A modified insurance risk process with uncertainty

Kai Yao[1], Zhongfeng Qin[2]

[1]*School of Management, University of Chinese Academy of Sciences, Beijing 100190, China*

[2]*School of Economics and Management, Beihang University, Beijing 100191, China*

yaokai@ucas.ac.cn, qin@buaa.edu.cn

Abstract

An insurance risk process is traditionally considered by describing the claim process via a renewal reward process and assuming the total premium to be proportional to the time with a constant ratio. It is usually modelled as a stochastic process such as compound Poisson process, and historical data are collected and employed to estimate the corresponding parameters of probability distributions. However, there exists the case of lack of data such as for a new insurance product. An alternative way is to estimate the parameters based on experts' subjective belief and information. Therefore, it is necessary to employ uncertain process to model the insurance risk process. In this paper, we propose a modified insurance risk process in which both the claim process and the premium process are assumed to be renewal reward processes with uncertain factors. Then we give the inverse uncertainty distribution of the modified process at each time. On this basis, we derive the ruin index which has an explicit expression based on given uncertainty distributions.

Keywords: Insurance risk process; Uncertain variable; Ruin; Ruin index; Uncertainty theory

1 Introduction

The Lundberg-Cramer insurance risk process was first proposed by Lundberg in 1903 and further developed by Cramer in 1930s. It assumes that the claims arrive according to a Poisson process with random compensations and the premiums are proportional to time with a constant ratio. The obtained Lundberg inequality and Lundberg-Cramer approximation provide the main results in ruin theory. Then Ammeter [3] and Andersen [4] generalized the conclusions to the case that the claim numbers follow a nonhomogeneous Poisson process or a general renewal process, respectively. Based on these works, the claim number process was further assumed to be an Erlang(2) process by Dickson and Hipp [9], and to be an Erlang(n) process by Li and Garrido [17], respectively. In addition, martingale was also applied to describe an insurance risk process by Gerber [12]. Except for ruin probability, the ruin time and deficit also draw some research interest in ruin theory. For example, Gerber and Shiu [13] studied the probability distribution of the deficit when an insurance company ruins, and Dickson and Hipp [10] studied the probability distribution of the ruin time of an insurance company.

With the in-depth research in ruin theory, inflation, dividend and tax are also considered to extend the insurance risk process. For example, Sundt and Teugels [28] presented a case with an constant interest rate, and Paulsen and Gjessing [25] proposed another one with dividends, which was further studied by Li and