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Comparative ambiguity aversion and downside ambiguity aversion

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ABSTRACT

This paper first defines an increase in ambiguity and an increase in downside ambiguity. We then provide comparative criteria for ambiguity aversion and downside ambiguity aversion. Different from the finding that the comparative criterion for risk aversion is variant with the measure of the premium to reduce risks, we show that the criteria remain the same, whether the premiums to reduce ambiguity and downside ambiguity are measured by utility or money. Under the criteria, a more ambiguity-averse (downside-ambiguity-averse) individual is shown to spend more effort in reducing ambiguity (downside ambiguity) than a less ambiguity-averse (downside-ambiguity-averse) individual.

1. Introduction

Ellsberg (1961) found that individuals tend to prefer a lottery with certain probabilities to a lottery with uncertain probabilities. Such a preference that cannot be explained by the theory of expected utility is known as ambiguity aversion. Since then, researchers have developed various models to characterize this ambiguity-averse preference, such as maxmin expected utility (Gilboa and Schmeidler, 1989), Choquet expected utility (Schmeidler, 1989), α-maxmin expected utility (Ghirardato et al., 2004), the smooth model of ambiguity aversion (Klibanoff et al., 2005), and so on.[1] Based on these models, fruitful findings regarding decision making under ambiguity aversion have been discovered.[2] On the other hand, some researchers have devoted to experimental studies and further enriched our understanding of ambiguity preferences.[3]

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1 Regarding the smooth model of ambiguity aversion (Klibanoff et al., 2005), its intertemporal version which is called recursive smooth ambiguity model (Klibanoff et al., 2009) and its dual representation (Iwaki and Osaki, 2014) have been developed. Furthermore, more general ambiguity models which accommodate the above models and relax their assumptions have also been proposed, such as Maccheroni et al.'s (2006) variational characterization, Siniscalchi's (2009) vector expected utility, Nau's (2011) state-preference theory, Cerreia-Vioglio et al.'s (2011) uncertainty averse representation, and Ghirardato and Siniscalchi's (2012) local and global multiple-prior characterization.

2 For instance, Gollier (2011) proved that under certain conditions, greater ambiguity aversion decreases the demand for the ambiguous asset but increases the equity premium. Snow (2011) showed that an increase in ambiguity aversion raises both the demand for self-insurance and the demand for self-protection. However, these results do not necessarily hold for more than two states of nature (Alary et al., 2013). Furthermore, Hoy et al. (2014) found that ambiguity aversion can help to explain the fact that few people are willing to receive a free genetic test.

3 For example, Cabantous (2007) found that insurers charge higher premiums in the presence of ambiguity than in the absence of ambiguity and that the charged premiums are higher when the insurers face with conflict ambiguity than with imprecise ambiguity. Cabantous et al. (2011) also discovered similar results, whereas when considering different types of risks, the result of higher charged premiums for conflict ambiguity than for imprecise ambiguity is reversed under the risks with abundant data such as fire risk. Charness et al. (2013) pointed out that social interactions affect ambiguity attitudes by inducing ambiguity loving and incoherent individuals to behave ambiguity neutrality. Moreover, from the

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In addition to characterizing the preference of ambiguity aversion, researchers have been exploring how to compare the degree of ambiguity aversion across individuals. For example, under the α-maxmin expected utility model (Ghirardato et al., 2004), the function *a*, which is the weight on the maxmin expected utility, is used to measure the degree of ambiguity aversion. An individual with a larger value of the function *a* behaves more pessimistically, indicating a more ambiguity-averse person. Under the smooth ambiguity aversion model (Klibanoff et al., 2005), one is said to be more ambiguity averse than the other when one's ambiguity function is obtained by the other's ambiguity function transformed with a strictly increasing and concave function, given the same utility function and subjective beliefs. Klibanoff et al. (2005) also proposed a measure for ambiguity aversion named the coefficient of ambiguity aversion, which is analogous to the Arrow–Pratt absolute risk aversion index.[4]

Beyond ambiguity aversion, third-order ambiguity attitude has not received much attention in the literature. One notable exception is the study of Baillon (2013). Analogous to risk apportionment (Eeckhoudt and Schlesinger, 2006), Baillon (2013) used the preferences over different options that vary in the probabilities of lotteries to define ambiguity attitudes, which is called ambiguity apportionment. In this approach, ambiguity aversion is defined as the preference of the option with certain probabilities of lotteries and third-order ambiguity attitude, which he called ambiguity prudence, is defined as the preference of the option in which the reduction and variation of the probabilities of lotteries are separated.[5] Furthermore, he showed that if the state space satisfies certain properties, under the smooth model of ambiguity aversion (Klibanoff et al., 2005), ambiguity aversion and ambiguity prudence correspond to an ambiguity function with a negative second derivative and one with a positive third derivative, respectively.[6] However, regarding the intensity of the third-order ambiguity attitude, as far as we know, no paper explores it.

This paper extends this line of the literature by providing comparative criteria for ambiguity aversion and downside ambiguity aversion under the smooth model of ambiguity aversion (Klibanoff et al., 2005). Compared with other models on ambiguity, the smooth ambiguity model is more suitable for our purpose because the preference-based definition of ambiguous events in its framework can disentangle non-constant ambiguity attitude and the ambiguity of an event (Klibanoff et al., 2011). In the literature, stochastic dominance approach has been a prevalent and powerful tool for ordering risks, characterizing risk preferences, and analyzing comparative statics regarding risks under the expected utility model. Given the success of stochastic dominance approach, for ambiguity, we develop similar notions by adopting the smooth ambiguity aversion model (Klibanoff et al., 2005) with a framework like the expected utility model.

experimental results, Conte and Hey (2013) found that among four types of models (the expected utility model, the smooth ambiguity aversion model, the rank dependent expected utility model, and the α-maxmin expected utility model), the smooth ambiguity aversion model has the best explanatory and predictive power for individuals' behavior in the presence of ambiguity.

4 Ahn (2008) proposed a model whose domain is the set of lotteries other than the state space. He proposed a index similar to that of Klibanoff et al. (2005).

5 This preference also represents the preference of the option with a less-spread left tail on the distribution of probabilities of lotteries, which is similar to the definition of downside risk aversion (Menezes et al., 1980). Thus, in this paper, we use the term *downside ambiguity aversion* for third-order ambiguity attitude.

6 In his paper, fourth-order ambiguity attitude (ambiguity temperance) and Nth-order ambiguity attitude were also defined and the sign of the Nth derivative of the ambiguity function could be determined by $(-1)^{n+1}$ if the state space satisfies certain properties.

Following Rothschild and Stiglitz (1970) and Ekern (1980), we first provide formal definitions of an increase in ambiguity and an increase in downside ambiguity. Then, we propose the conditions under which one is more ambiguity averse or downside ambiguity averse than the other. These conditions are analogous to those proposed by Ross (1981) and Modica and Scarsini (2005), respectively, for risk aversion and downside risk aversion. Furthermore, we find that the conditions of comparative ambiguity aversion and downside ambiguity aversion defined by the monetary premium are the same as those defined by the utility premium. This finding is different from the results on risks in the literature, such as those of Jindapon and Neilson (2007).[7]

As an application of the comparative criteria, we study the problem of spending efforts in reducing ambiguity and downside ambiguity which is related to but different from what Huang (2012) has examined. In the presence of ambiguity, we study the impact of more ambiguity aversion on optimal efforts to reduce ambiguity while Huang (2012) studied its impact on optimal efforts to reduce risks. Moreover, our more ambiguity aversion measure is developed in the spirit of Ross's (1981) comparative risk aversion measure while Huang's (2012) measure is defined as Klibanoff et al.'s (2005) absolute ambiguity aversion measure which was proposed in the spirit of the Arrow–Pratt risk aversion measure. We show that an individual will spend more efforts in reducing ambiguity (downside ambiguity) if and only if he or she is more ambiguity averse (downside ambiguity averse) defined by our criteria. This finding is analogous to the result of Jindapon and Neilson (2007), where efforts are spent to reduce risks.

Our paper contributes to the literature in three ways. First, this paper is, to our best knowledge, the first to offer definitions of an increase in ambiguity and an increase in downside ambiguity for general ambiguous probability distributions. In contrast to the Bernoulli distribution studied by Snow (2011) and Alary et al. (2013), for general probability distributions, the notion of an increase in ambiguity or in downside ambiguity depends crucially on how the unknown parameter in the probability distribution is defined. To use the mean-preserving spread and the mean–variance-preserving spread to describe the increase in ambiguity and the increase in downside ambiguity, respectively, the unknown parameter must be properly specified. Second and moreover, we develop comparative ambiguity aversion and downside ambiguity aversion in the spirit of Ross (1981). This opens up a new way to analyze the intensity of higher-order ambiguity attitudes that complements the apportionment approach adopted by Baillon (2013) when studying higher-order ambiguity attitudes. Third, we clarify that conditions for comparative ambiguity aversion and downside ambiguity aversion are indifferent whether the premiums to reduce ambiguity and downside ambiguity, respectively, are measured by utility or money. This finding identifies a key difference between comparative ambiguity aversion and comparative risk aversion.

The rest of our paper is organized as follows. Section 2 provides the definitions for an increase in ambiguity and an increase in downside ambiguity. Section 3 develops the notion of comparative ambiguity aversion and downside ambiguity aversion, while Section 4 presents an application in spending efforts in reducing ambiguity and downside ambiguity. We conclude the paper in Section 5 and relegate all proofs to the Appendices.

7 In general, conditions of comparative risk aversion defined by the monetary premium lead to Ross's (1981) measure of risk aversion, whereas conditions of comparative risk aversion defined by the utility premium lead to the Arrow–Pratt measure of risk aversion.