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Dividend optimization for regime-switching general diffusions

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HIGHLIGHTS

- We study dividend optimization for a regime-switching general diffusion model.
- The optimal strategy, if exists, is a regime-switching barrier strategy.
- It is possible that an optimal strategy does not exist.

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ABSTRACT

We consider the optimal dividend distribution problem of a financial corporation whose surplus is modeled by a general diffusion process with both the drift and diffusion coefficients depending on the external economic regime as well as the surplus itself through general functions. The aim is to find a dividend payout scheme that maximizes the present value of the total dividends until ruin. We show that, depending on the configuration of the model parameters, there are two exclusive scenarios:

(i) the optimal strategy uniquely exists and corresponds to paying out all surpluses in excess of a critical level (barrier) dependent on the economic regime and paying nothing when the surplus is below the critical level;
(ii) there are no optimal strategies.

1. Introduction

As a classical topic in finance and actuarial science, the dividend optimization problem is termed as "the management's problem of determining the optimal timing and sizes of dividend payments in the presence of bankruptcy risk". The general framework in addressing this problem in the literature is to use a diffusion process or a compound Poisson process to model the evolution of the surplus of a corporation in the absence of dividend payments (uncontrolled process) and then subtract the dividend payments from the current surplus, which leads to the controlled surplus process. The management of the company can dynamically control the dividend payments and its objective is to find the optimal dividend payout scheme so that the expected total discounted value of the dividend payments until ruin is maximal.

The dividend optimization problem has been solved for diffusion models with constant drift and diffusion coefficients (Asmussen and Taksar, 1997; Taksar, 2000). There is a wealth of literature considering the extensions of the optimal dividend problem under the same framework by incorporating the risk control measures such as investment and reinsurance, debt liability, transaction costs, and other factors; see Asmussen et al. (2000), Taksar (2000), Guo et al. (2004), Yang et al. (2005), Cadenillas et al. (2006), He and Liang (2009) and the references therein.

A more general and more realistic assumption for the diffusion models is that the drift and diffusion coefficients of the

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"uncontrolled" surplus process vary as the surplus itself varies. Some justifications for this are that in reality the surplus can generate interest and thereby alters the drift coefficient of the surplus and that the current level of surplus will affect the return rate and variability of future revenues. Højgaard and Taksar (2001) assumed that the surplus can be invested in a financial asset with the price process governed by a geometric Brownian motion, which gives rise to an uncontrolled surplus process that follows a diffusion with the drift and diffusion coefficients proportional to the current surplus and its square root, respectively. In Bäuerle (2004) the surplus was assumed to earn interest at a constant rate and hence the drift coefficient is a function of the surplus while the diffusion coefficient is still a constant. A mean-reverting process was used to model the uncontrolled surplus process in Cadenillas et al. (2007). A general diffusion model where the drift and diffusion coefficients are both functions of the surplus was used to model the uncontrolled surplus process in Shreve et al. (1984), Paulsen (2003), Alvarez and Virtanen (2006) and Paulsen (2007, 2008).

Another direction of generalizing the models for the uncontrolled surplus process is to allow the evolution of the surplus to depend on the external economic regime. As argued, e.g. by Jiang and Pistorius (2012), Markov regime-switching process models are appropriate to describe the macroeconomic quantities, which is supported by a substantial literature in econometrics. Models with regime switching have been used by Zhu and Yang (2008, 2009) in studying the ruin related problems of insurance companies. Sotomayor and Cadenillas (2011) considered a diffusion process with both the drift and diffusion coefficients modulated by a continuous Markov chain with two states (regimes) but independent of the level of the surplus, and solved the dividend optimization problem for two cases: bounded dividend payment rate and unbounded dividend rate. The dividend optimization problem for the Markovian regime switching diffusion process with multiple (more than two) regimes and unbounded dividend rates has been solved by Jiang and Pistorius (2012), where the drift and diffusion coefficients only depend on the regime but not the surplus. A Markov regime switching diffusion process with the drift and diffusion coefficients proportional to the surplus was used by Cajueiro and Yoneyama (2004) in studying the optimal portfolio and consumption problem for a small agent. Sotomayor and Cadenillas (2009) considered the consumption–investment problems in regime switching diffusion models where the drift and diffusion coefficients are affine functions of the surplus. The dividend optimization problem under the compound Poisson model with regime switching has been studied in Yin et al. (2010) and Wei et al. (2010).

In this paper, we attempt to solve the dividend optimization problem under a diffusion process model for the uncontrolled surplus where the drift and diffusion coefficients are allowed to depend on both the surplus itself and the external economic regime through general functions. We show that either there is a unique optimal dividend strategy, which is a regime switching barrier strategy, or there are no optimal strategies, dependent on the model configuration.

The significance of this paper is that the model we study generalizes the diffusion type models considered in the literature, and better describes the real evolution of the surplus. Our model is general enough to include most of the diffusion models considered in the literature as special cases. The complex nature of the model at this level of generality makes it seemingly impossible to solve the associated dividend optimization problem using the commonly used approaches. We manage to solve the optimization problem under the general diffusion model by studying a few relevant auxiliary operators first instead of solving it directly. Our results regarding the optimal strategy not only unify the existing results for similar types of controls under different diffusion models to some extent, but also can serve as a guideline for the management.

The rest of the paper is organized as follows. In Section 2 we introduce the Markov regime-switching diffusion model for the surplus and formulate the optimal dividend problem as an optimal stochastic control problem. In Section 3 we define the regime switching barrier strategy and introduce some relevant operators. We study the properties and the relationships between the operators, by which important properties of the value function V are derived. The solution to the optimization problem is also presented in this section. In Section 4 we present numerical examples to illustrate the feasibility of numerically implementing the developed solutions and to appreciate the effects of model parameters on the solutions. Section 5 concludes with some remarks.

2. Problem formulation

Consider a complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathrm{P})$, where $\{\mathcal{F}_t\}_{t\geq 0}$ is a right-continuous filtration. Let $\{W_t; t \geq 0\}$ be a standard Brownian motion relative to $\{\mathcal{F}_t\}_{t\geq 0}$ and $\{J_t; t \geq 0\}$ be an $\{\mathcal{F}_t\}_{t\geq 0}$- Markov chain independent of the process $\{W_t; t \geq 0\}$. Assume that the state space of the Markov chain is $E = \{1, 2, \ldots, m\}$ and the intensity matrix $Q = (q_{ij})_{m\times m}$. For notational convenience, we write $q_i = -q_{ii} = \sum_{j\neq i} q_{ij}$ for $i \in E$. In our context, E represents the collection of economic regimes, and J_t indicates the economic regime at time t.

Let X_t denote the surplus at time t in the absence of dividend payments. Assume X_t follows the following dynamics

$$\mathrm{d}X_t = \mu(X_{t-}, J_{t-})\mathrm{d}t + \sigma(X_{t-}, J_{t-})\mathrm{d}W_t, \tag{2.1}$$

where, for any fixed $i \in E$, the functions $\mu(\cdot, i) : \mathbb{R}^+ \to \mathbb{R}^+$ and $\sigma(\cdot, i) : \mathbb{R}^+ \to \mathbb{R}^+$ are differentiable, Lipschitz continuous and grow at most linearly. Furthermore, we assume that $\mu(\cdot, i)$ is concave with $\mu(0, i) > 0$, that there exists an $M > 0$ such that $\mu'(x, i) \leq \delta_i$ for $x \geq M$, and that $\sigma(\cdot, i)$ is non-vanishing.

Now assume that the company also pays dividends. Let L_t denote the accumulated dividends paid up to time t. It is reasonable to assume that $L = \{L_t; t \geq 0\}$ is nondecreasing with $L_0 = 0$, left-continuous with limits from the right, and adapted to $\{\mathcal{F}_t\}_{t\geq 0}$. Following the literature, we identify the accumulated dividend process L with the underlying dividend distribution strategy. Incorporating dividend payouts, the dynamics of the (controlled) surplus process with strategy L is described by the following stochastic differential equation

$$X_t^L = \int_0^t \mu(X_{s-}^L, J_{s-})\mathrm{d}s + \int_0^t \sigma(X_{s-}^L, J_{s-})\mathrm{d}W_s - L_t. \tag{2.2}$$

The time to ruin of the company is defined to be the time when the controlled surplus first becomes negative,

$$T = \inf\{t \geq 0 : X_t^L < 0\},$$

which, by the path properties of the Brownian motion, equals the time when X_t^L first hits 0,

$$T = \inf\{t \geq 0 : X_t^L \leq 0\}.$$

Remark 2.1. Since the process L is nondecreasing and left continuous, it has the following decomposition: $L_t = L_t^c + \sum_{0\leq s<t}(L_{s+} - L_s)$, where $\{L_t^c; t \geq 0\}$ is the continuous part of the process $\{L_t; t \geq 0\}$. Furthermore, $L_{t+} = L_t^c + \sum_{0\leq s\leq t}(L_{s+} - L_s)$.

A dividend strategy L is *admissible* if $L_t \equiv L_T$ for $t \geq T$ and $L_{t+} - L_t \leq X_t^L$ for $t \geq 0$. In other words, an admissible strategy pays no dividends after ruin and it does not allow an instant dividend payment to be bigger than the current level of surplus. For convenience, we use X and X^L to denote the processes $\{X_t; t \geq 0\}$ and $\{X_t^L; t \geq 0\}$, respectively. We use $\Pi(X)$ to denote the collection of all admissible strategies for the (uncontrolled) surplus process X.